

Tractable Staggered Bargaining

Previously “The Nash-in-Kalai Model for Estimation with Dynamic Bargaining and Nontransferable Utility”

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Staggering is Everywhere

- Anchor tenants commit to renting for decades
- YouTube TV-Disney exclusion due to previous MNFs
- Healthcare price multiples in place for 10+ years

But all of our empirical models assume simultaneous bargaining

(Horn and Wolinsky, 1988; Crawford and Yurukoglu, 2012; Grennan, 2013; Lee and Fong, 2013; Gowrisankaran et al., 2015; Ho and Lee, 2017, 2019; Ghili et al., 2023; Crawford et al., 2018; Tiew, 2022; Barwick et al., 2025)

This Paper

1. When reality is staggered, our options are:
 - Assume simultaneity and introduce bias
 - Allow staggering and introduce forward-looking state space growth
2. The step-by-step property controls state space growth
 - Kalai proportional yields tractable moment that nests TU Nash-in-Nash
3. In a real-world application to healthcare, accurate timing matters
 - Static model yields $\hat{\tau} = -7.54$; myopic gets right weights but wrong dynamics

Will Show Bias with Two Staggered Two-Period Contracts

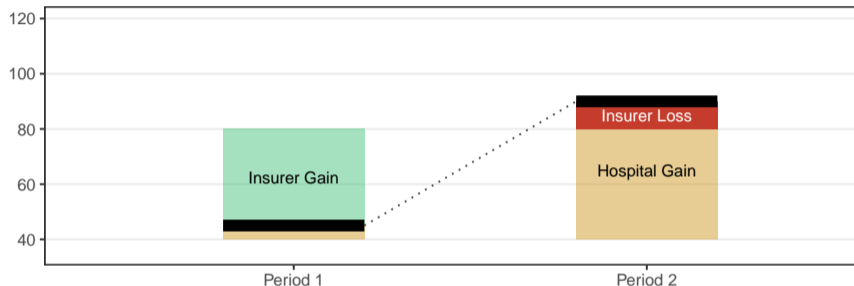


Figure: Contracts are a stylized version of list price-based negotiations. The joint NPV gains for discount factor $\beta = 0.5$ are $80 - 40 + 0.5 \times (80 - 40) = 60$. If $p_2 = 2p_1$, then $p_1 = 45$ gives the hospital $(45 - 40) + 0.5 \times (90 - 40) = 30$, $1 - \tau = 1/2$ of the joint gains.

Incorrect Period-by-Period Models Add Bias

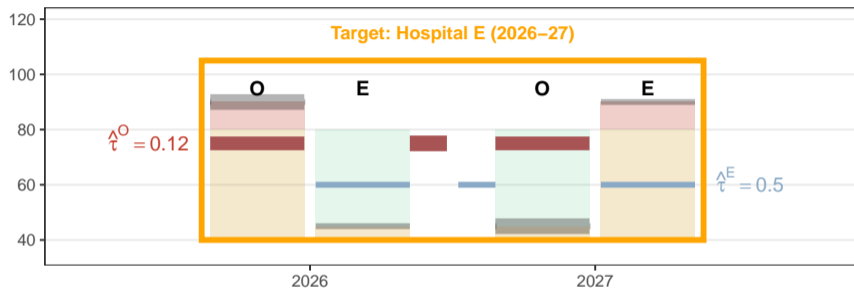


Figure: An incorrect period-by-period model will introduce bias. Hospital *E* negotiates in 2026, and hospital *O* negotiates in 2027 (and 2025 off-screen). Fundamentally, no fixed time aggregation can specify correct GFTs for all staggered contracts. Natural solution: explicitly model staggering!

Staggered Bargaining Yields Explosive State Space Growth

- Under staggering, every remaining length in the market is a relevant state
 - Contracting externalities: p_E affects insurer's leverage with O (Lee et al., 2021)
 - Staggering: $p_{E,t} \rightarrow p_{E,t+1}$ affects insurer's leverage with O next year
 - Forward-looking: every remaining length shows up in the bargaining problem
- Staggering yields explosive state growth
 - 9 pairs with 5-year contracts in a monthly model \Rightarrow 1.6 quadrillion price-response functions
- Can a model of forward-looking bargaining avoid the explosive state space growth introduced by staggered contracts?

Illustration: State Space Cancellation Under Step-by-Step

- Step-by-step, loosely: $\tau GFT_2 = (1 - \tau)GFT_1$ in every subgame [▶ Animation](#)
- Two-period agreement (V^A) negotiated relative to negotiating tomorrow (βV^{D1})
- Tomorrow's agreement (V^{D1}) negotiated relative to the day after (βV^{D2})

$$\tau(V_2^A - \beta V_2^{D1}) = (1 - \tau)(V_1^A - \beta V_1^{D1})$$

$$\tau(V_2^{D1} - \beta V_2^{D2}) = (1 - \tau)(V_1^{D1} - \beta V_1^{D2})$$

Can eliminate D1: $\tau(V_2^A - \beta^2 V_2^{D2}) = (1 - \tau)(V_1^A - \beta^2 V_1^{D2})$




State Space Control in Multilateral Settings

Number of price-response functions for the 9-pair, 5-year contract, monthly model:

- In principle: 1.6 quadrillion
- Under step-by-step (worst case): 3.5 billion
- Under step-by-step (best case): 9

And the worst case will have a much simpler approximation

I Study Nash-in-Kalai, Which Maintains Step-by-Step

- Nash satisfies step-by-step under simultaneous TU, loses under staggering 
- Kalai proportional: uniquely satisfies in general (Kalai, 1977; Roth, 1979) 
 - Justified via monotonicity (Kalai, 1977), identification (Dorn, 2026), or revocation costs (Dutta, 2012, 2024)
 - Same as Nash under TU (Lee and Fong, 2013; Collard-Wexler et al., 2019; Ho and Lee, 2017; Crawford et al., 2018)
- I propose Nash-in-Kalai: Nash equilibrium in Kalai proportional bargains
 - Nests TU Nash-in-Nash, but keeps step-by-step tractability advantages beyond TU
 - I offer a non-TU microfoundation  \Rightarrow strictly more microfounded than Nash-in-Nash

But how is Nash-in-Kalai for use in practice?



The Nash-in-Kalai Moment Enables GMM Estimation


$$E_{t_0} \left[\sum_{t=t_0}^{t^*} \beta^{t-t_0} \text{Pay}_{ijt}^* \right] = E_{t_0} \left[\sum_{t=t_0}^{t^*} \beta^{t-t_0} \text{Pay}_{NiN,ijt} \right] + E_{t_0} \left[\sum_{t=t_0+1}^{\infty} \beta^{t-t_0} \text{Pay}_{DA,ijt} \right]$$

- NPV Kalai payment = NPV Nash-in-Nash flow payments + dynamic adjustment
- Pay_{DA} corrects for how ij disagreement shifts non- ij future prices
 - $\text{Pay}_{DA} = 0$ in steady state \Rightarrow nests existing TU estimation strategies
 - Can capture $\text{Pay}_{DA} \neq 0$ with bounded rationality approximation (level-1, level-2, ...)

In data, does the static approach's misspecified timing introduce bias?

Empirical Application: Hospital–Insurer Contracting in WV

- West Virginia made hospital–insurer contracts public records 
- Results draw on companion paper Dorn (2025)
 - 2006–2015 panel, 63 estimation bargains across six insurers and 27 hospital systems
- Smaller insurers/larger hospitals: more prices based on fast-growing list prices 

Are contracts multiyear (\Rightarrow inflation) and staggered (\Rightarrow potential bias)? Yes. 

Empirical Model: Ho and Lee (2017) + Dynamics

- Hospitals–insurer bargaining over multiperiod contracts using Kalai ▶ Timing
- Ho and Lee (2017) flow profits with **price externalities** and $\underline{\beta}$ (annual) ≥ 0 :

$$\pi_j^{Ins} = D_j^{Ins}(\cdot)(\phi_j - \eta_j) - \sum_{h \in \mathcal{G}_j^{Ins}} D_{hj}^{Hosp}(\cdot) p_{hj} \text{ and } \pi_i^{Hosp} = \sum_{n \in \mathcal{G}_i^{Hosp}} D_{in}^{Hosp}(\cdot)(p_{in} - c_i)$$

- Three specifications — all imply Ho and Lee (2017) moment + NPVs ▶
 1. Static: 2015 payments, treated as negotiated in 2015
 2. Myopic: Accurate timing, but impose $\beta = 0$
 3. Forward-looking: Accurate timing and allow $\beta > 0$ under Kalai

Key Implementation Details for $\beta > 0$

- Approximations: level-1 bounded rationality & five-year finite horizon
 - Can probe in future work, but at least more general than anything before
- β identified from list price-based contracts' faster price inflation + τ_{ij} restrictions

Static Model's Misspecified Timing Yields Incoherent Weights

		Parameter			
	β	τ_{BCBS}	τ_{HPUOV}	τ_{FP}	$-\tau^{Size}$
Static	.	0.487**	-7.54	0.694***	3.354
	(.)	(0.191)	(17.204)	(0.175)	(22.875)
Myopic	.	0.876***	0.825***	0.861***	1.037***
	(.)	(0.012)	(0.232)	(0.034)	(0.199)
Forward-Looking	0.899***	0.854***	0.877***	0.889***	0.989***
	(0.03)	(0.006)	(0.026)	(0.005)	(0.028)

Note:

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Table: The static model predicts wrong and incoherent weights due to τ^{Size} colinearity [▶ \$\tau^{Size} = 0\$](#) .
 The myopic model gets weights right, but gets forward-looking responses wrong. [▶ Shares](#) [▶ More](#)

Conclusion

- We can write down a forward-looking model that stays tractable with staggering
 - Dorn (2025) applies to Medicare dynamics, will apply to other dynamics like list prices
- Empirical results: static model's misspecified timing can introduce bias
 - Here: need accurate timing for bargaining weights, forward-looking for dynamic responses
- New open questions like predicting timing bias in settings with unobserved timing

Thoughts welcome! `jacobdorn@cornell.edu`

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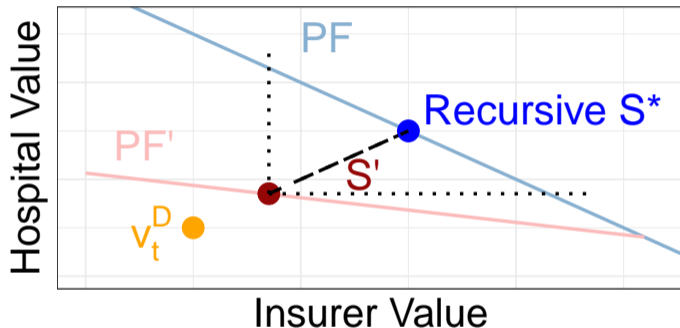
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The Step-by-Step Property



Loosely, GFTs always solve $\tau(V_2^A - v_2^D) = (1 - \tau)(V_1^A - v_1^D)$ ◀

Why Nash Loses Step-by-Step

$$\text{Nash share: } \frac{V_1(p^*) - v_1^D}{V_1(p^*) - v_1^D + V_2(p^*) - v_2^D} = \frac{\tau V_1'(p^*)}{\tau V_1'(p^*) - (1 - \tau)V_2'(p^*)}$$

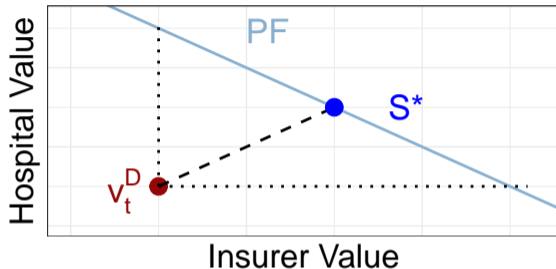
$$\text{Step-by-step: } \frac{V_1(p^*) - v_1^D}{V_1(p^*) - v_1^D + V_2(p^*) - v_2^D} = \tilde{\tau} \quad \text{Fixed.}$$

- If staggered price is $p_{h,t} = p^* + \lambda_1(p_{-h,t-1} - p^*) \Rightarrow \text{Nash share} = \tau + \frac{\tau(1-\tau)\beta\lambda_1}{1+\tau\beta\lambda_1}$
- But in off-equilibrium bargain, $V_1'(p^*) = -V_2'(p^*) \Rightarrow \text{Nash share} = \tau$
- Nash only satisfies step-by-step in homothetic games

Aside: What/Why is this Kalai Proportional Bargaining?

$$\frac{GFT_t^{Ins}(p_{Kalai}^*)}{GFT_t^{Hosp}(p_{Kalai}^*)} = \frac{\tau_{ij}}{1 - \tau_{ij}}$$

$$\frac{GFT_t^{Ins}(p_{Nash}^*)}{GFT_t^{Hosp}(p_{Nash}^*)} = \frac{\tau_{ij}}{1 - \tau_{ij}} \underbrace{\frac{-V_t^{Ins}(p_{Nash}^*)'}{V_t^{Hosp}(p_{Nash}^*)'}}_{=1 \text{ if TU}}$$



- Static view: “central role in the theory” (Thomson, 1994) — axioms, intuition, and data
- Generalizes Nash for **transferable utility** models like Ho and Lee (2017), but I show important tractability advantages for NTU dynamics

◀ Empirical model

Nash-in-Kalai Microfoundation

- Big Kalai downside: scale-varying, so cannot microfound via vNM utility alone
- I offer a microfoundation for staggered non-TU Nash-in-Kalai
 - Uses Dutta (2012)'s delegate revocation costs to fix utility scales
 - Nash-in-Nash is only microfounded for simultaneous TU (Collard-Wexler et al., 2019)
 - Corollary: Nash-in-Kalai is now strictly more microfounded than Nash-in-Nash

Public record contract report panel

- Data from 2006–15 West Virginia
- Scans of annual hospital reports [▶ Example](#)
- Use for payment rates, networks, benchmarks, timing, ...
- Other data: inpatient discharges (2016), state-level premiums & sales (2006–16)

Example West Virginia Contract Report Scan

Discount Contract List Budgeted Discounts for FY 2016 Hospital Name Charleston Surgical Hospital

Name of Third Party Payor	Inpatient %	Outpatient %	Inpatient	Outpatient
1 C&O Employees (auto-renewal)	N/A	6 00% ✓	Must Separate	Combine
2 Select-Net (auto-renewal)	10 00% ✓	10 00% ✓	Combine	Combine
3 Cigna (auto-renewal)	18 00% ✓	15 00% ✓	Combine	Combine
4 4Most (auto-renewal)	5 00% ✓	5 00% ✓	Combine	Combine
5 MDI (auto-renewal)	15 00% ✓	10 00% ✓	Combine	Combine
6			Combine	Combine

List discounts in lower section that are (1) new or not currently approved contracts, (2) non-third party (e.g. admin. adj.), (3) contracts with utilization > calculated volume threshold above*, (4) HMO or risk contracts, or, (5) top section of template determined that it must be separated

1 Mt State-PPO	43 38%	41 58%	Must Separate	Must Separate
2 Mt State-Indemnity	43 38%	38 45%	Must Separate	Must Separate
3 Aetna	18 00% ✓	15 00% ✓	Must Separate	Must Separate
4 Carelink	15 00% ✓	13 00% ✓	Must Separate	Must Separate
5 United	10 00% ✓	10 00% ✓	Must Separate	Must Separate

Figure: Charleston Surgical Hospital report, fiscal year 2016. Mountain State/Highmark BCBS generally used Medicare as a benchmark (non-round numbers) while other smaller insurers generally used list prices. [▶ Was WV Unrepresentative? ◀](#)

West Virginia Rate Regulation

- From 1993-2016, West Virginia:
 1. Capped hospital charge increases
 2. Required all hospital–insurer contracts to cover average costs
 3. Approved hospital–insurer contracts and made them public records
- Does this make West Virginia unrepresentative?
 - 1 & 2: Caps “too generous” as of Murray and Berenson (2015) and contracts easily covered costs, though may have been associated with lower list prices and more outpatient care
 - 3: disclosure unusual at time — may be more representative of where the US is going

Medicare-Based Benchmarks Associated with Bigger Insurers

Insurer	Medicare	List Prices
All	46.74	53.26
Modeled	60.20	39.80
Highmark BCBS	72.27	27.73
HPUOV	56.24	43.76
Other Modeled	13.14	86.86
Nonmodeled	3.03	96.97

Table: Estimated percentage of 2011-16 projected inpatient payments classified as Medicare-benchmarked and list price-benchmarked. [▶ Algorithm](#) [◀](#)

How I Infer Benchmarks

- Share-of-charges: same reported % of charges (up to 0.01%) in consecutive years
- Prospective (likely Medicare-based): anything else
- Possible overestimate: include per diems, any non-Medicare DRG formulas
- Possible underestimate: more charge usage than other settings (Cooper et al., 2019; Weber et al., 2019)



How to Think About Contracts:

Multiyear and formed at different times

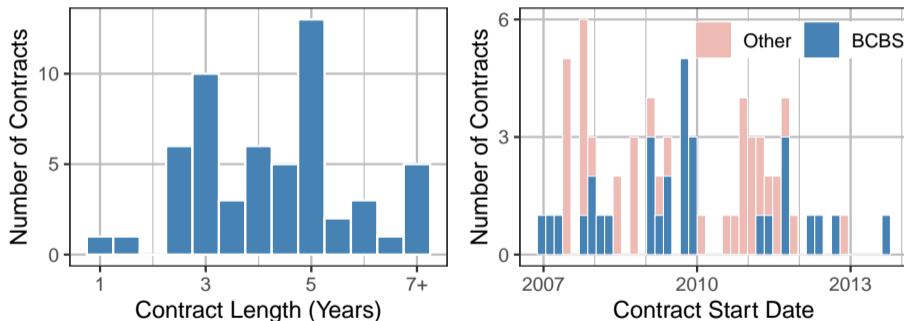


Figure: Histograms of BCBS reported lengths (left) and contract formation dates (right).

▶ Others



Non-BCBS Contract Lengths (Auto-Renew)

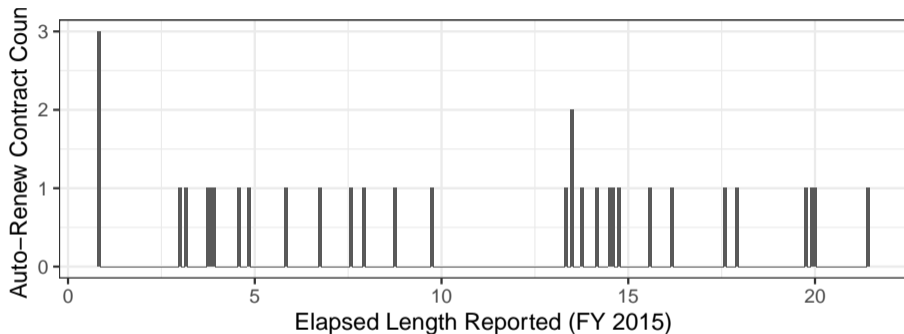



Figure: Retrospective length for non-BCBS modeled insurer auto-renew contracts (where available) as of fiscal year 2015. ◀


Empirical Model: Ho and Lee (2017) + Dynamics

1. Renewed-contract prices are updated (background process)
2. Hospitals and insurers simultaneously bargain new contracts
 - Contracts can last more than one period — annual discounting factor β
 - Use Kalai solution to tractably capture forward-looking dynamics
3. Consumers choose plans and get sick \Rightarrow hospital, insurer demand D^{Hosp}, D^{Ins}
4. Flow profits realized with **price** externalities — some internalized 

$$\pi_j^{Ins} = \underbrace{D_j^{Ins}(\cdot)(\phi_j - \eta_j)}_{\text{Premium revenue net of } \eta} - \underbrace{\sum_{h \in \mathcal{G}_j^{Ins}} D_{hj}^{Hosp}(\cdot) p_{hj}}_{\text{Payments to hospitals}} \quad \text{and} \quad \pi_i^{Hosp} = \underbrace{\sum_{n \in \mathcal{G}_i^{Hosp}} D_{in}^{Hosp}(\cdot)(p_{in} - c_i)}_{\text{Payments received - cost of care}}$$

Shared Estimation Moment

$$0 = E_{t_0} \left[\sum_{t=t_0}^{\min\{t^*, t_0+4\}} \beta^{t-t_0} \left(D_{ijt}^H P_{ijt} - \overbrace{\left((1 - \tau_{ij}) [\Delta_{ij} \pi_{jt}^M] - \tau_{ij} [\Delta_{ij} \pi_{it}^H] \right)}^{\text{Nash-in-Nash flow payment}} \right) \right]$$

- Ho and Lee (2017) moment + NPVs: $[\Delta_{ij} \pi]$ is effect of ij tie on pre- Pay_{ij} profits
- Insurer weight τ_{ij} varies by insurer (τ_j) and hospital system size (τ^{Size}) 
- β identified from list price-based contracts' faster price inflation
 - Instruments: insurer dummies & hospital-size-bin dummies

Bargaining Weight Formula

$$\log(\tau_j/(1 - \tau_j)) + \tau^{Size} \log(HospSize_{i,2006}/MeanHospSize_{2006}) = \log(\tau_{ij}/(1 - \tau_{ij})) =$$

- τ_j fixed effects: BCBS, HPUOV, or for-profit evaluated at mean hospital system
- $\tau^{Size} < 0$: larger hospitals have more bargaining weight
- Hospital size evaluated in 2006 (start of data) to avoid endogeneity

Discount Factor Identification From Benchmarks


- Suppose we see a two-year agreement with $c^H = 10$ & $v^M = 50$ constant
 - Bargain to split NPV gains $(50 - 10) + \beta \times (50 - 10)$ with $\beta = 1/2$ unknown
 - Observe $p_1 = 20$ and $p_2 = 50$; insurer share is known to be $\tau = 1/2$
- GMM recovers β via $E[p_1 - c + \beta(p_2 - c) - (1 - \tau)GFT_1 - \beta(1 - \tau)GFT_2] = 0$
 - $\beta = 0$ implies $1 - \tau = \frac{10+0 \times 40}{40+0 \times 40} = \frac{1}{4} \Rightarrow$ data rejects $\beta = 0$
 - $\beta = 1/2$ implies $1 - \tau = \frac{10+0.5 \times 40}{40+0.5 \times 40} = \frac{30}{60} = \frac{1}{2} \Rightarrow \hat{\beta} = \frac{1}{2}$
- Need $p_2/p_1 - GFT_2/GFT_1$ variation: use list prices vs. Medicare by firm
- Since τ is estimated, identify from restrictions on τ_{ij} heterogeneity

Bargaining Parameter Estimates, No Size Heterogeneity

	Parameter				
	β	τ_{BCBS}	τ_{HPUOV}	τ_{FP}	$-\tau^{Size}$
Static	. (.)	0.365*** (0.011)	0.278* (0.143)	0.16*** (0.048)	. (.)
Myopic	. (.)	0.863*** (0.006)	0.845*** (0.016)	0.631*** (0.027)	. (.)
Forward-Looking	0.714*** (0.032)	0.852*** (0.012)	0.86*** (0.008)	0.685*** (0.028)	. (.)

Note:

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Table: Estimated bargaining parameters with τ^{Size} enforced to zero. The static model substantially underestimates for-profit insurers' weight, possibly due to long-lived list price multiples. 

Estimated Gain From Trade Shares (Will Change)

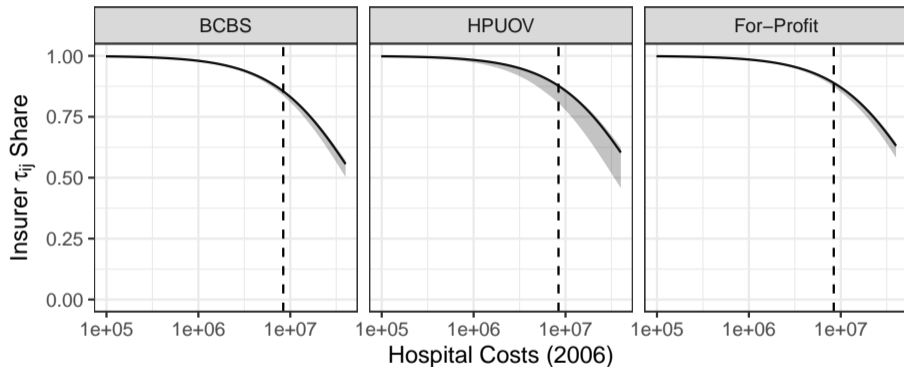


Figure: Share of gains from trade retained by the insurer under estimated forward-looking model.

► Myopic ◀

Estimated Gain From Trade Shares (Myopic)

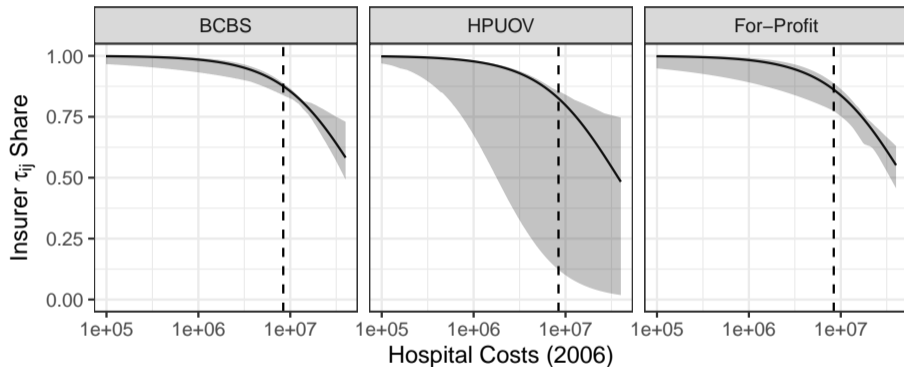


Figure: Estimated percent of gains from trade retained by the insurer under estimated myopic model. ◀

Other Bargaining Parameters

	Parameter (τ^{Size} Estimated)							
	η_{BCBS}	η_{HPUOV}	η_{Aetna}	$\eta_{UnitedHealth}$	η_{Cigna}	$\eta_{Carelink}$	r_{yBCBS}^M	r_{nBCBS}^M
Static	3657*** (45)	3404*** (85)	3658*** (116)	2008*** (29)	4627*** (32)	3139*** (39)	10000*** (2614)	9999*** (1441)
Myopic	4640*** (14)	4036*** (650)	3659*** (37)	3197*** (374)	4624*** (26)	3139*** (463)	10000*** (1444)	10000*** (1)
Forward-Looking	4638*** (130)	3631*** (302)	3660*** (37)	3284*** (69)	4626*** (30)	3140*** (45)	9999*** (29)	9999*** (65)
Data	3600	3356	3554	1999	4635	3114		

Note:

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Table: Estimated non-inpatient costs (η) and bargaining cost contributions (r). η parameters are partially fit to moments from MLR: the “Data” values minimize the MLR moment. r parameter estimates stay at their initial value, which suggests they may not be identified. 