The Nash-in-Kalai Model for Estimation with Dynamic Bargaining and Nontransferable Utility

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Current Dynamic Models Rule Out Non-TU Mechanisms

- Empirical dynamic bargaining models are transferable utility (TU)
  - Dynamic: multiperiod agreements, recursive gains, or uncertainty
  - Transferable utility: at the margin, my dollar gained is your dollar lost
- TU rules out real-world nontransferable utility (NTU) effects
  - IO: simultaneous contracts — see staggering in data
  - Macro-labor: employers match offers — do not always in practice

This paper: how to model dynamic nontransferable utility (NTU) bargaining?

Today's Talk

1. Two agents in a single-period, uncertain world
   Nash weights unidentified, Kalai proportional weights can be identified
2. Two agents in a multiperiod, recursive world
   Simpler equivalent disagreement from step-by-step utility property
3. Multiple agents in a multiperiod, uncertain, recursive, world
   Nash-in-Kalai: GMM under even nonstationary NTU dynamics

NTU + Uncertainty $\Rightarrow$ Nash Weights Unidentified

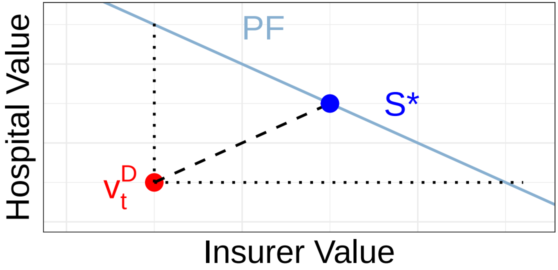
Goal: a setting where Nash weight τ or $\overbrace{\tau'}^{\leq \tau \ \mathit{WLOG}}$ can imply $p^* = 0\tau'$

- Bargain price p with $u_1(p) = \tau' X^r - p$ and $u_2(p) = 1 + \frac{X^{-r}}{1-\tau'} p$, $X \sim \mathit{Unif}([0,1])$
- Can find r such that Nash weight τ applied before X is realized yields $p^* = 0$
- By scale-invariance, Nash weight τ' applied after X is realized always has $p^* = 0$

I Use the Kalai Proportional Solution

$$\frac{GFT_t^{Ins}(p^*_{Kalai})}{GFT_t^{Hosp}(p^*_{Kalai})} = \frac{\tau_{ij}}{1-\tau_{ij}}$$

$$\frac{GFT_t^{Ins}(p^*_{Nash})}{GFT_t^{Hosp}(p^*_{Nash})} = \frac{\tau_{ij}}{1-\tau_{ij}} \underbrace{\frac{-V_t^{Ins}(p^*_{Nash})'}{V_t^{Hosp}(p^*_{Nash})'}}_{=1 \text{ if TU}}$$

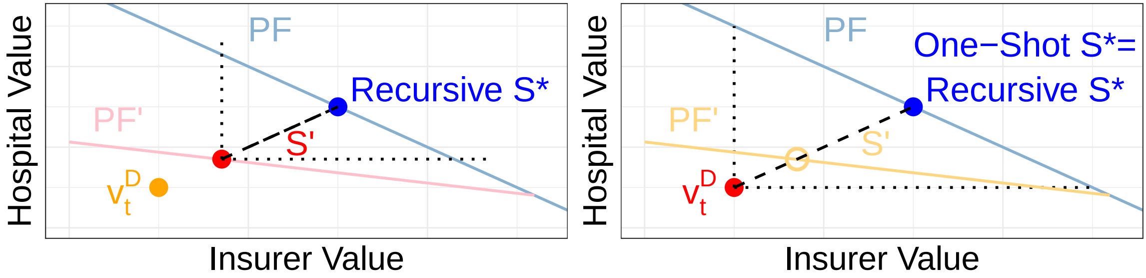


- Static view: "central role in the theory" (Thomson, 1994) — axioms, intuition, and data
- Same as Nash if TU, I show uniquely enables IIA + identification if NTU

How to Characterize Bargaining in a Multiperiod World?

- Choose a T-period agreement (A) relative to (D1) outcome
- (D1) recursively Kalai bargained relative to (D2) and so on
- (D1), (D2), etc. depend on future conditions in a messy way

Step-by-Step will Simplify Recursive Bargaining



- *Step-by-step* property (Kalai, 1977), AKA *path independence* (Roth, 1979)
- Kalai (1977): "One has to be careful," not a condition "on the underlying game"

How Step-by-Step Simplifies Recursive Bargaining

- Agreement (A) chosen relative to (D1) chosen relative to (D2)
- Step-by-step: can replace (D1) with (D2) and get same (A)
  - Induction $\Rightarrow$ can use some (DT) with finite dependence-type simplification
  - Yields sufficient conditions for nonparametric identification
- In continuous time, can use ($D\infty$): *impasse* in discrete time

Next: an empirically tractable framework for multiple firms interacting

The Nash-in-Kalai Model for Multilateral Interactions

1. Information revealed $+$ non-bargaining strategic interactions
2. Simultaneous bargaining over new agreements
   - Kalai proportional bargaining over expected NPV
   - Recursive: bargain in terms of future strategies
   - Shared rational expectations and discounting rate β
3. Flow profits and negotiation costs

But how to handle recursive disagreement? Equivalent (D∞) impasse point

Nash-in-Kalai Impasse $\approx$ Nash-in-Nash Disagreement

- Recursive Kalai: surprise disagreement in one period
  - A priori plausible, but massive relevant state space
- Static Nash: surprise disagreement in world's single period
  - Tractable, but does not extend to a multiperiod world
- Impasse Kalai: surprise disagreement in every period
  - Extends static Nash tractability to a multiperiod world
  - Equivalent to recursive Kalai by step-by-step

Nash-in-Kalai Yields a Moment for GMM Estimation

Theorem 1'

Suppose ij negotiate in period t_0 ▸ and some assumptions hold. Then $E[\text{NPV Payment}_{ij}]$ is:

$$\mathbb{E}_{t_0}\left[\overbrace{\sum_{t=t_0}^{t^*}\beta^{t-t_0}Pay_{ijt}}^{\text{NPV Payment}}\right]=\mathbb{E}_{t_0}\left[\sum_{t=t_0}^{t^*}\beta^{t-t_0}\overbrace{\text{Pay}_{NiN,ijt}}^{\substack{\text{Static Nash}\\\text{Flow Payment}}}+\overbrace{\text{Pay}_{NC}+\text{Pay}_{IRT}}^{\substack{\text{Added Multiperiod}\\\text{Bargaining Terms}}}\right],$$

where Pay_{NC} reflects negotiation costs and Pay_{IRT} reflects the effect of spillovers on impasse profits ▸ Form, which is zero in steady state. ▸ Empirical application

- Extends myopic Nash: $\beta = 0$

This Paper

- Nash weights unidentified with NTU uncertainty $\Rightarrow$ use Kalai
- Step-by-step to simplify recursive Kalai bargaining
- Nash-in-Kalai for GMM under dynamics + strategic interactions
- (Also, there is an empirical application)

Feedback welcome! `jdorn@upenn.edu`

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Identification in Single-Period Games $\Leftrightarrow$ Kalai

Theorem 1

Suppose $\mathcal{F}$ is a family of bargaining solutions satisfying independence of irrelevant alternatives that can explain any Pareto-efficient outcome. Then $\mathcal{F}$ is identified with general u_1, u_2 if and only if $\mathcal{F}$ is the set of Kalai proportional solutions.

- Key concept is concavity: everyone weakly prefers ex ante negotiation
- Myerson (1981): concavity + IIA implies proportional and/or utilitarian
- I show identification implies concavity, not utilitarian-only

Two Sufficient Conditions for N.P. Identification when $T > 1$

Regularity conditions are not enough — also one of...

1. No unobserved utility ($\varepsilon = 0$), yes variation in agreements $\mid X$
   - Inspired by a comment by Rust (1994)
   - Correlation of $(p^*, x') \mid x$ must reflect information & identify β
2. Exclusion: $Z \subseteq X$ only affects p^* through $E[p'/p]$
   - Often feasible in practice
   - Identification of β from $Cor(Z, p^* \mid X)$ and $Cor(Z, p'/p \mid X)$

Two-Agent (Non)Identification Regularity Conditions

- Markov strategies, perfect (well-calibrated) expectations, $E[\varepsilon] = 0$
- A static-type instrument such that $E[\text{hypothetical flow payment} \mid X]$ identifies τ

Equivalent Static Representation

- Suppose $\beta > 0$ and ε is always equal to zero
- Take $\beta' = 0$, $u_i'(x) = E[V(C^*, x) - V(\text{Disagreement}, x)]$
- Same (constant) payments, but different utility and $\hat{\tau}$

Other Assumptions (High Level)

- Risk-neutral, shared unbiased expectations, shared discount rate $\beta \in [0, 1)$
- Markov strategies, good-faith disagreement (no change in networks)
- Negotiation cost paid to validate a successful bargain (for Pay_{NC})

Kalai Relative to Impasse

1. *Kalai bargaining*. *ij* always choose contracts from Kalai proportional bargaining over expected values with fixed weight weight τ_{ij}
2. *Impasse disagreement*. *i* and *j* disagreement value: everyone will reach contracts expecting *ij* agreement, but *i* and *j* will ultimately fail to reach an agreement
3. *Discrete time*. No bargaining is attempted between equilibrium bargaining times

Impasse Repricing Transfer

If A and D subscripts denote the path of prices and premiums under agreement-then-impasse and immediate impasse, the Impasse Repricing Transfer is:

$$
\begin{aligned}
Pay_{IRT} = {} & \mathbb{E}_{t_0}\left[\sum_{t=t_0+1}^{\infty} \beta^{t-t_0}(-\tau_j) \sum_{n \in \mathcal{G}_{it}^{Hosp}/j} \left(\begin{array}{rl} & D_{int}^{Hosp}\left(\mathcal{G}_t/ij, \phi_{jt|t_0}^{A}\right)\left(p_{int|t_0}^{A} - c_i\right) \\ - & D_{int}^{Hosp}\left(\mathcal{G}_t/ij, \phi_{jt|t_0}^{D}\right)\left(p_{int|t_0}^{D} - c_i\right) \end{array} \right)\right] \\
& + \mathbb{E}_{t_0}\left[\sum_{t=t_0+1}^{\infty} \beta^{t-t_0}(1-\tau_j) \left(\begin{array}{rl} & D_{nt}^{Ins}(\mathcal{G}_t/ij, \phi_{t|t_0}^{A})(\phi_{jt|t_0}^{A} - \eta_j) \\ - & D_{nt}^{Ins}(\mathcal{G}_t/ij, \phi_{t|t_0}^{D})(\phi_{jt|t_0}^{D} - \eta_j) \end{array} \right)\right] \\
& + \mathbb{E}_{t_0}\left[\sum_{t=t_0+1}^{\infty} \beta^{t-t_0}(1-\tau_j) \sum_{h \in \mathcal{G}_{jt}^{Ins}/i} \left(\begin{array}{rl} & D_{hjt}^{Hosp}(\mathcal{G}_t/ij, \phi_{t|t_0}^{A}) p_{hjt|t_0}^{A} \\ - & D_{hjt}^{Hosp}(\mathcal{G}_t/ij, \phi_{t|t_0}^{D}) p_{hjt|t_0}^{D} \end{array} \right)\right].
\end{aligned}
$$

Empirical Application

- Adapted from Dorn (2025): hospital–insurer bargaining in West Virginia
- See contracts are multiyear and staggered $\Rightarrow$ forward-looking bargaining is NTU
- Identify β from predictable differences in benchmark inflation by firm
- Estimate $\beta = 0.899$ (reject myopia), and find static model would get τ wrong