The Nash-in-Kalai Model for Estimation with Dynamic Bargaining and Nontransferable Utility

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Abstract

This paper extends empirical models of dynamic bargaining to allow for the possibility of nontransferable utility. The model agrees with Nash bargaining when utility is transferable, but leverages the Kalai proportional bargaining solution to enable identification in the presence of uncertainty over nontransferable Pareto frontiers. The model extends the class of dynamic bargaining models for which there is a known valid general method of moments estimation strategy to include agreements that have foreseeable and nontransferable contracting externalities on the distribution of future states, and the model uniquely possesses a discrete-time representation even when bargaining is defined in continuous time. I estimate an empirical model in this class using novel hospital–insurer contract panel data from West Virginia, where agreements had nontransferable contracting externalities on other deals reached while the terms remained in place. I find that dynamic aspects of bargaining are important: the estimated annual discounting rate of 0.899 corresponds to a strong forward-looking response.

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1 Introduction

Prevailing empirical models of bargaining are either static, transferable utility (TU), or both. In static models, the world is one-period and negotiators know all states at which any bargain will take place. In TU models, negotiators possess an instrument (typically price) that helps one party as much as it harms the other side. TU models allow the Nash bargaining solution to be derived by first choosing an agreement that maximizes bilateral surplus, and then choosing a utility transfer to achieve desired surplus shares.

In dynamic settings, TU corresponds to a strong, and often unrealistic, restriction that prices do not affect any future negotiation. For example, in industrial organization, TU precludes staggered contracting, because a price negotiation between a retailer and a supplier could otherwise have a foreseeable and nontransferable externality on how the retailer negotiates with other suppliers while the contract remains in place. In labor economics, TU requires employers to fully match outside offers, because initial wages would otherwise have a nontransferable externality on with the employee's post-departure income. The restrictions imposed to achieve TU can rule out important aspects of bargaining, but dynamic nontransferable utility (NTU) Nash bargaining models are generally limited to theory (Do and Miklós-Thal, 2022) or calibration (Gertler and Trigari, 2009; Gottfries, 2022).

The first contribution of this paper is to show that when bargaining involves potential uncertainty over NTU Pareto frontiers, Nash bargaining weights are unidentified. For any bargaining weights $\tau_1, \tau_2 \in (0, 1)$, I provide a distribution of feasible utilities for which the outcome of choosing a price ex ante through Nash bargaining with weight τ_1 is observationally equivalent to the outcome of choosing a price ex post through Nash bargaining with weight τ_2. As a result, Nash bargaining weights are not identified unless the econometrician knows whether bargaining occurs ex ante or ex post. I show that identification for this class of games is only possible under a timing property that Myerson (1981) calls concavity. Myerson shows that in social choice problems, very few solution concepts satisfy independence of irrelevant alternatives (IIA) and concavity. By extending his characterization to bargaining problems, I show that only one bargaining solution satisfying IIA enables identification: the Kalai (1977) proportional bargaining solution.

The second contribution of this paper is to propose a new bargaining model, which I call the *Nash-in-Kalai* bargaining model. The model is a Nash equilibrium in recursively-defined Kalai proportional bargains over expected net present values. The Kalai proportional solution maximizes joint surplus, subject to choosing an agreement along the ray of utility pairs that achieve the desired surplus shares. In the model, gains from trade are calculated taking as given past outcomes and current strategies, but are recursively defined in terms of future negotiations that may be uncertain and influenced by the terms of the current contract. The solution concept agrees with Nash-in-Nash bargaining in TU games, because the agreement achieving the maximum surplus along the desired-surplus-share ray (Kalai proportional bargaining) is the agreement maximizing joint surplus with a utility transfer to achieve desired surplus shares (Nash bargaining). This paper characterizes some unique advantages of Kalai proportional bargaining in dynamic NTU games.

I derive sufficient conditions for nonparametric identification of Kalai proportional bargaining parameters. When agreements are single-period but potentially uncertain, Kalai proportional bargaining weights are identified, but Nash bargaining weights are unidentified. This is because under Kalai proportional bargaining, both ex ante and ex post negotiators choose transfers that achieve the same shares of gains in expectation, but under Nash bargaining, the realized Pareto frontier influences ex post negotiators' share of gains from trade. With multiperiod agreements, the forward-looking Nash-in-Kalai model is unidentified due to the existence of an observationally equivalent static representation. To achieve nonparametric identification with multiperiod agreements, I propose two different sufficient conditions. First, there is identification if there are no unobserved components of utility and there is variation in agreements conditional on the observed states. In this case, the discounting rates is identified from the association of future states and starting prices. Alternatively, there is identification if an instrument satisfies an exclusion restriction of only affecting value functions through inflation expectations. In this second case, the discounting rate can be identified from the instrument's joint association with future inflation and starting prices.

The proof of multiperiod identification relies on a convenient equivalent representation of the Nash-in-Kalai solution. Under recursive Kalai proportional bargaining, negotiators (and the econometrician) can replace the value of one disagreement with the value of repeated disagreement. This simplification is especially valuable for empirical work when disagreement may be short-lived, as the value of repeated disagreement, which Binmore et al. (1989) call the *impasse point*, is uniquely defined in discrete time even as disagreement becomes arbitrarily short. This dynamic advantage corresponds to a static utility property that Kalai (1977) calls *step-by-step*, and which is essentially unique to Kalai proportional bargaining.

The third contribution of this paper is to show that Nash-in-Kalai bargaining provides a moment on expected net present value (NPV) transfers, enabling general method of moments (GMM) estimation of bargaining parameters. The expected transfer is the sum of the expected NPV of Nash-in-Nash flow transfers, a transfer to split negotiation costs, and a transfer to reflect the any difference between the Nash-in-Nash disagreement point, which reflects disagreement at future equilibrium prices, and the Nash-in-Kalai impasse point, which reflects disagreement at the prices that others form under impasse. This expected NPV transfer coincides with the Nash-in-Nash prediction in the case of TU bargaining, but the Nash-in-Kalai moment holds whether bargaining is TU or NTU, whether uncertainty is resolved before or after bargaining, and whether the equilibrium is stationary or nonstationary. Econometric errors can reflect either expectational error from uncertainty or unobserved components of utility, facilitating GMM estimation with appropriate instruments.

The main restrictions for the Nash-in-Kalai moment are that negotiators must share a common discounting rate and unbiased beliefs about future conditions. These assumptions are substantive, but often plausible in settings in which the leading sources of uncertainty are shared. Further, these conditions are required to derive a consistently-defined moment on expected NPV transfers: two sides need to share a discounting rate to unambiguously map a path of transfers to a NPV, two sides need symmetric beliefs in order to agree on the expected NPV transfer, and the beliefs must be accurate in order for the econometrician to use realized transfers in GMM estimation.

I estimate an empirical model in this class using a public record administrative dataset from West Virginia on hospital–insurer contracts, which in other settings have been considered trade secrets. Critically, the data shows that contracts were staggered, with realized terms running from three years to more than one decade. Staggered contracts are only consistent with TU bargaining under myopia. If negotiators are forward-looking about staggered contracts, then they will foresee the NTU externality of an initial contract term with the market state at both sides will negotiate with others while the contract remains in place. My empirical model and estimation strategy generally follow the GMM approach in Ho and Lee (2017), but I allow negotiators to balance multiple periods of profits. The discounting rate β is identified by the predictable larger effect of increasing β on NPV transfers for firms with longer-lived contracts and faster price growth.

The estimated model clearly rejects a static view of the world. The staggered contracts reject a literally static model. The estimated annual discounting rate of $\beta = 0.899$ rejects the null hypothesis of myopia needed to maintain TU bargaining, and implies that expected future conditions have an important role in shaping current prices. I also compare the estimated bargaining parameters with the results of a static Nash-in-Nash model that only uses one year of payments as an input, and assumes that all payments reflected recently-formed agreements. I find that the static Nash-in-Nash approach would underestimate small insurers' bargaining weight, likely because smaller insurers' high current prices partially reflect accumulated price growth under long-lived contracts.

The Nash-in-Kalai model proposed here is an extension of the TU Nash-in-Nash bargaining model. I follow Lee et al. (2021)'s excellent introduction to this literature and refer to the effect of a contracted term on other strategic decisions as a "contracting externality." Most empirical Nash-in-Nash work employs a static model as an approximation to a true dynamic process. The TU case includes many static empirical models of American healthcare and telecommunications (Ho and Lee, 2017, 2019; Crawford et al., 2018), the microfoundations of static Nash-in-Nash (Collard-Wexler et al., 2019), and the smaller literature on dynamic Nash-in-Nash (Lee and Fong, 2013; Liu, 2021; Tiew, 2022; Deng et al., 2023). There is empirical work on static NTU bargaining, particularly in models in which a negotiated upstream price has a direct and nontransferable effect on quantity supplied (Grennan, 2013; Gowrisankaran et al., 2015; Ghili, 2022), and theoretical work on dynamic NTU Nash-in-Nash bargaining in triangular markets (De Fraja, 1993; Bárcena-Ruiz and Casado-Izaga, 2008; Do and Miklós-Thal, 2022).[1] The Nash-in-Nash and Nash-in-Kalai models do not coincide for NTU bargaining, but a hypothetical dynamic NTU Nash-in-Nash model would have similar predictions to a dynamic Nash-in-Kalai model if bargaining is close to TU, or more generally when the dynamic Nash solution predicts an approximately-fixed bilateral surplus share over time.

The leading case of dynamic bargaining models for observational data involve wage negotiations. Models of search-on-the-job generally impose a TU bargaining game or a split-the-surplus outcome (Cahuc et al., 2006; Bagger et al., 2014; Bilal et al., 2022; Jarosch et al., 2024). Gertler and Trigari (2009) and Gottfries (2022) propose models of NTU wage bargaining in stationary settings, which require calibration for empirical practice. I provide most identification results in the context of two agents bargaining on their own, so the associated theoretical advantages of the Kalai proportional solution concept apply to wage negotiation after slight adjustments to the empirical model.

This work also contributes to the theoretical literature on multilateral solution concepts. Kalai (1977) and Roth (1979) show that the step-by-step property for static utility is essen-

[1]An aside on notation. I follow the coalitional bargaining literature and use "NTU" to refer to the generalization of TU bargaining problems to asymmetric or nonlinear Pareto frontiers (Mas-Colell and Hart, 1996). In matching problems, "NTU" is used to refer to extreme cases in which no compensation is possible, and "imperfectly transferable utility" is used to refer to cases that this paper would call NTU (Galichon et al., 2019). The matching NTU definition rules out bargaining, so I use the more general coalitional NTU definition.

tially unique to Kalai proportional bargaining. I show that the step-by-step property is a valuable state simplification property in dynamic games, and yields Binmore et al. (1989)'s impasse point as an equivalent disagreement value. My results for (non)identification with single-period agreements relate to Myerson (1981)'s result that among social choice functions that satisfy IIA, only utilitarian end egalitarian solutions satisfy concavity. Relative to Myerson's uniqueness result, I show that concavity is a necessary condition for identification with potential uncertainty, show that scale-invariance (as in Nash bargaining) is incompatible with concavity, extend uniqueness to bargaining problems, and note that utilitarian bargaining weights are only partially identified.

My results for identification in multiperiod agreements also relate to the literature on (non)identification in dynamic discrete choice models (Abbring, 2010), in particular the nonidentification results of Rust (1994) and Magnac and Thesmar (2002). My first proposed condition for nonparametric identification is motivated by a remark by Rust (1994). My second condition is motivated by Magnac and Thesmar (2002)'s proposed identification from an exclusion condition in dynamic discrete choice problems. In dynamic discrete choice problems, the first set of conditions is known to be insufficient for identification (Rust, 1994) and the second condition has no natural analog.

The plan of the paper is as follows. Section 2 reviews some key concepts for bargaining in utility space, proves that Nash bargaining weights are unidentified in the presence of NTU bargaining with possible uncertainty, and shows that Kalai proportional bargaining is the unique solution satisfying IIA and identification in single-period games. Section 3 extends the model to consider infinite-length games with two agents, shows that Kalai bargaining identification carries over with single-period but not multiperiod agreements, illustrates the step-by-step property for simplifying recursive Kalai proportional bargaining, and provides two different sufficient conditions for identification with multiperiod agreements. Section 4 further extends the model to consider many agents negotiating agreements at different times that interact, and derives a moment on expected NPV transfers. Section 5 describes my empirical application to hospital–insurer bargaining in West Virginia. Section 6 concludes.

2 Starting Point: Bilateral Bargaining in a Single-Period World

To most readers, the Nash-in-Kalai model will suggest several questions:

(1) What is Kalai proportional bargaining?

(2) Why is Kalai proportional bargaining only sometimes the same as Nash bargaining?

(3) Why is an alternative to Nash bargaining needed for dynamic games? and

(4) Why might Kalai proportional bargaining be the right alternative?

I answer the first two questions by reviewing some key concepts for static bargaining in utility space. I answer the last two questions with new results on identification for two agents negotiating over a price to apply for the world's single period.

2.1 What is Kalai Proportional Bargaining?

The Kalai proportional bargaining solution is a solution concept for Nash's bargaining problem.

A Nash bargaining problem G (for game) is a closed set S of feasible agreement values in $\mathbb{R}^2$ and a disagreement value $v^D \in S$. I use "$\geq$" and "$>$" to refer to the pointwise comparison: $s \geq s'$ if and only if $s_1 \geq s'_1$ and $s_2 \geq s'_2$, and analogously $s > s'$ means both sides strictly prefer s to s'. I will make a few regularity assumptions on S.

Assumption 1 (Regularity conditions)**.** The family of games $\mathcal{G}$ satisfies convexity (for all $s_1, s_2 \in S$ and $\lambda \in [0,1]$, $\lambda s_1 + (1-\lambda)s_2 \in S$) and comprehensiveness (if $s \in S$ and $v^D \leq s' \leq s$, then $s' \in S$).

Convexity is an important regularity condition for bargaining games (Shimer, 2006), and generally reduces a bargaining problem to a Pareto frontier and disagreement value. Comprehensiveness ensures that Kalai proportional bargaining is well-defined (Roth, 1979). The comprehensive convex hull of $\{s_1, \ldots, s_k\}$ is the set of $s \in \mathbb{R}^2$ such that $s \leq \sum_j \lambda_j s_j$ for some nonnegative λ_j summing to one (Myerson, 1981).

Within a bargaining game, there are many ways of choosing an allocation that makes both sides better off than disagreement. A particular bargaining solution is a function $f : \mathcal{G} \to \mathbb{R}^+$ such that $f(S, v^D) \in S$ and $f(S, v^D) \geq v^D$. I write that $f' = f$ on $\mathcal{G}$ if for all $(S, v^D) \in \mathcal{G}$, $f(S, v^D) = f'(S, v^D)$, and shorthand $f' = f$ if the two are equal for all games or $\mathcal{G}$ is clear in context.

Definition 1 (Kalai proportional bargaining)**.** *The* Kalai proportional bargaining solution *with player-1 weight* $\tau \in [0, 1]$ *is* $f_\tau^{(Kalai)}(S, v^D) = (\max t : (t\tau, t(1-\tau)) \in S)\,(\tau, 1-\tau)$.

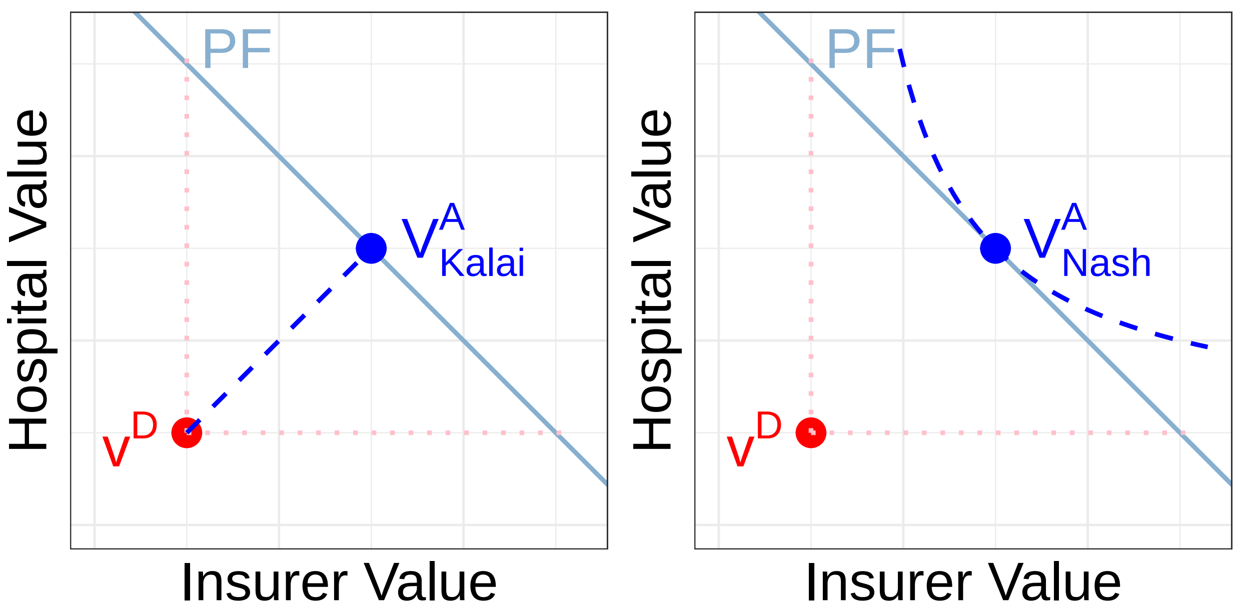

Figure 1: Left: the Kalai proportional bargaining solution for bargaining between an insurer (x axis) and hospital (y axis) with Pareto frontier PF in light blue and disagreement value v^D in red, and Pareto constrained indicated by dotted lines. The desired-surplus-split ray $v = tp$ is indicated by the dashed blue line, and the resulting agreement is indicated by v^A_{Kalai}. Right: the Nash bargaining solution to the same game. The dashed line indicates the Nash bargaining indifference curve, and the resulting agreement is indicated by v^A_{Nash}.

The Kalai proportional solution for bargaining between a hospital (player 2) and insurer (player 1) is illustrated in Figure 1(a). I draw a dashed ray to indicate the values $v \in S$ that yield positive gains from trade and the desired surplus split $\tau(v_2 - v_2^D) = (1-\tau)(v_1 - v_1^D)$. The solution V_{Kalai}^A is the best agreement along this desired-surplus-split ray.

2.2 Why is Kalai Bargaining Only Sometimes the Same as Nash Bargaining?

Definition 2 (Nash bargaining)**.** *The Nash bargaining solution with player-one bargaining weight* $\tau \in [0,1]$ *is* $f_\tau^{(Nash)}(S, v^D) = \text{argmax}_{v \in S} \left(v_1 - v_1^D\right)^\tau (v_2 - v_2^D)^{1-\tau}$.

The Nash bargaining solution is illustrated in Figure 1(b). The Nash bargaining solution can be viewed as maximizing a joint utility function, with player one receiving greater weight. The joint utility function generates a joint indifference curve. In well-behaved problems with negotiations over a price, the Nash bargaining first-order condition at the agreed price p_{Nash}^* can be rewritten as:

$$\tau v_1'(p_{\text{Nash}}^*)(v_2^* - v_2^D) = -(1-\tau) v_2'(p_{\text{Nash}}^*)(v_1^* - v_1^D).$$

Kalai and Nash bargaining will have the same predictions when bargaining is TU: if the Pareto frontier is a line segment with a constant slope of negative one.

Assumption 2 (Transferable utility)**.** A bargaining game family satisfies Assumption 1 and each element satisfies *transferable utility* if S is the comprehensive convex hull of v^D, $v^D + (v, 0)$, and $v^D + (0, v)$ for some $v > 0$.

Equivalence follows by algebra.

Lemma 1 (Nash and Kalai coincide for TU games)**.** *Suppose* $\mathcal{G}$ *satisfies Assumption 2. Then* $f_\tau^{(Nash)} = f_\tau^{(Kalai)}$ *on* $\mathcal{G}$*.*

Proof. Let $G \in \mathcal{G}$ be given. For s on the Pareto frontier of S, write $p(s) = \frac{s_2 - v_2^D}{\bar{v}_2 - v_2^D}$. Then $v_2^* + v_1^* = v_2^D + v_1^D + v$ and

$$\tau v_1' \left(p\left(f_\tau^{(Nash)}(S, v^D)\right)\right)(v_2^* - v_2^D) = \frac{\tau(f_\tau^{(Nash)}(S, v^D)_2 - v_2^D)}{\bar{v}_2 - v_2^D}$$
$$= (1-\tau) v_2' \left(p\left(f_\tau^{(Nash)}(S, v^D)\right)\right)(v_1^* - v_1^D) = \frac{(1-\tau)(f_\tau^{(Nash)}(S, v^D)_1 - v_1^D)}{\bar{v}_2 - v_2^D},$$

so that $\tau(v_2^* - v_2^D) = (1-\tau)(v_1^D + v - v_2^* - v_1^D)$. But $f_\tau^{(Kalai)}$ satisfies the same constraint. Therefore $f_\tau^{(Nash)}(S, v^D) = f_\tau^{(Kalai)}(S, v^D)$. □

Some important advantages of Nash bargaining are that it is weakly Pareto-optimal and satisfies independence of irrelevant alternatives and scale-invariance (Serrano, 2005).

Definition 3 (Bargaining properties)**.** *For $a \in \mathbb{R}^d$ for $d = 1$ or 2, let $aS = \{as : s \in S\}$. A bargaining solution f is (i)* weakly Pareto-optimal *(WPO) if for all S, v^D, there is no $s \in S$ such that $s > f(S, v^D)$; (ii) satisfies* independence of irrelevant alternatives *(IIA) if $T \subseteq S$ and $f(S) \in T$ implies $f(T, v^D) = f(S, v^D)$; and (iii) is* scale-invariant *if for all $a \in \mathbb{R}_2$ such that $a > 0$, $f(aS, av^D) = af(S, v^D)$.*

WPO is a minimal condition for a bargaining solution. IIA is a necessary condition for f to maximize some notion of utility (Myerson, 1981).[2] Scale invariance implies that if one modifies the scale of one side's gains from trade by a fixed constant, then the other side's outcome is unaffected. I slightly abuse notation and write that if f satisfies IIA, S satisfies Assumption 1, and $f(S, v^D) \in T \subseteq S$, then $f(T, v^D) = f(S, v^D)$ even if T is not comprehensive.

Kalai proportional bargaining is not scale invariant, so it can only coincide with Nash bargaining within a family of games like TU that implicitly define a scale of relative utility. Evidence from the lab suggests the negotiators can be scale-varying in monetary games (Nydegger and Owen, 1974; Duffy et al., 2021). Two important conditions which Kalai

[2]The IIA axiom can be replaced by lower-level conditions that are outside the scope of this work (Thomson and Myerson, 1980).

proportional bargaining solutions and not Nash bargaining solutions satisfy are concavity and step-by-step, which I discuss later.

This completes the review of static bargaining in utility space. I now move to consider dynamic games, beginning with dynamic games that result in one-period agreements.

2.3 Why is an Alternative to Nash Bargaining Needed?

I now show that for any $\tau, \tau' \in (0,1)$, there is a distribution of utility functions for which ex ante bargaining over a price to apply to the uncertain utility functions under $f_{\tau}^{(Nash})$ and ex post Nash bargaining over a price to apply to the realized utility functions under $f_{\tau'}^{(Nash)}$ both produce a constant price of zero. As a result, Nash bargaining weights can only be identified with knowledge of whether negotiations occur before or after the realization of uncertainty. Empirical dynamic models are often applied to settings with information that is realized over time, making it unappealing to require the econometrician to take a substantive stance on unobservable information timing just to identify bargaining weights.

Consider the following class of games:

1. If $EA = 1$ (for ex ante), the players have the opportunity to negotiate over a price $p \in \mathbb{R}$ using bargaining solution f.

2. The game type R is drawn from a distribution P and revealed to the players.

3. If $EA = 0$, the players have the opportunity to negotiate over a price $p \in [0, 1]$.

4. If the sides have an agreement ($A = 1$), then player 1 receives utility $u_1(p \mid R)$ and player 2 received utility $u_2(p \mid R)$. If the sides have no agreement ($A = 0$), then both receive $u_i^D = 0$. I assume that for all R, there is a $p \in \mathbb{R}$ producing strict gains for both sides, i.e. with $u(p \mid R) > 0$.

5. The econometrician observes the distribution P, the game type R, the functions u_1 and u_2, and the price p^*.

I require some notation for showing that bargaining weights, and more generally bargaining solutions, are not identified. This section stays in a single-period world, so for now, a structure b is a value of EA, a distribution P, utility functions u, and strategies σ such that σ is the output of applying f to the generated bargaining problem. Note that a WPO bargaining solution generally achieves a Nash equilibrium, so that when f is a Nash bargaining solution, this is a Nash-in-Nash model (Lee et al., 2021), and when f is a proportional bargaining solution, this is a Nash-in-Kalai model, which I formalize later. I abuse notation and write $f(b)$ for the bargaining solution under b.

I write that b and b' are observationally equivalent if they generate the same observables: u_1, u_2, P and the joint distribution of (R, p^*). I shorthand observational equivalence as $b \Leftrightarrow b'$. I write that bargaining solutions f are equivalent if for every b with the bargaining solution f, there is a b' with the same EA, P, u, the bargaining solution f', and some σ' such that $b' \Leftrightarrow b$. In this case, I write $f \Leftrightarrow f'$.

Definition 4 (Single-period identification). *Let $\mathcal{F}$ be a family of bargaining solutions. I write that $\mathcal{F}$ is* single-period identified *if $b \Leftrightarrow b'$ implies $f(b) \Leftrightarrow f(b')$. I write that $\mathcal{F}$ is* single-period unidentified *if $\mathcal{F}$ is not single-period identified.*

If $\mathcal{F}$ is not single-period identified, then an econometrician who mistakenly conjectures that bargaining is ex post ($EA = 0$) can infer a bargaining solution that has the wrong counterfactual prediction even with known information structure.

For these single-period games, Nash bargaining weights can only be identified with knowledge of EA: it is possible to construct a game for which incorrectly conjecturing EA yields to an arbitrarily wrong estimate of the Nash bargaining weight τ.

Proposition 1 (Single-period Nash nonidentification). *Let $\mathcal{F}$ be a family of bargaining solutions that include $f_\tau^{(Nash)}$ and $f_{\tau'}^{(Nash)}$ for some distinct $\tau, \tau' \in (0,1)$. Then $\mathcal{F}$ is single-period unidentified.*

Proof. Without loss of generality suppose $\tau' \geq \tau$. Let $s = \sqrt{1 - \frac{\tau'(1-\tau)}{(1-\tau')\tau}} \in [0,1)$. Take

$u_1(p \mid r) = \tau r^s - p$, $u_2(p \mid r) = 1 + \frac{r^{-s}}{1-\tau}p$, $P = Unif(0,1)$, $f' = f_{\tau'}^{(Nash)}$, and $f = f_{\tau}^{(Nash)}$. Note that $E[R^s] = 1/(1+s)$ and $E[R^{-s}] = 1/(1-s)$. Let b be a structure generated by $(EA = 0, f)$, let b' be a structure generated by $(EA = 1, f')$, and let b'' be the structure generated by $(EA = 0, f')$.

In b, by inspection, every realized bargaining game satisfies Assumption 1. By scale invariance, the negotiated price is the price predicted by a game with player 2 gains instead $(1-\tau)r^g + p$, so that under b, $p^* = 0$ almost surely.

In b', the ex ante gains are $\frac{\tau}{1+s} - p$ and $1 + \frac{1}{(1-\tau)(1-s)}p$, respectively. This produces an equivalent price to a gain with gains $\frac{\tau}{1+s} - p$ and $(1-\tau)(1-s) + p$, respectively, which by inspection generates $p^* = 0$. Therefore $b \Leftrightarrow b'$.

But $b \not\Leftrightarrow b''$, so $f \not\Leftrightarrow f''$ and $\mathcal{F}$ is single-period unidentified. □

The proof uses only the scale-invariance property of Nash bargaining, and so it extends to any family that includes scale-invariant bargaining solutions with distinct interior predictions for TU games.

In dynamic games, it is useful to be able to identify bargaining weights without specifying exactly when information is observed. As a result, I propose going beyond Nash bargaining.

2.4 Why is Kalai Proportional Bargaining the Right Alternative for Dynamic Games?

Kalai proportional bargaining turns out to be the only bargaining solution that can identify bargaining weights for games in this class while satisfying IIA.

Assumption 3 (IIA and span)**.** $\mathcal{F}$ is a family of bargaining solutions that satisfy IIA; are distinct in the sense that no separate $f, f' \in \mathcal{F}$ satisfy $f = f'$ for all G generated by single-period games; and span all Pareto-efficient outcomes in the sense that if $s > v^D$ is Pareto optimal in some S, then there is an $f \in \mathcal{F}$ such that $f(S, v^D) = s$.

The span assumption is that the model is always able to infer a bargaining weight from a Pareto-efficient agreement. This assumption holds for families of bargaining solutions that are amendable to empirical use.

Theorem 1 (IIA and single-period identification implies Kalai proportional bargaining)**.** *Suppose $\mathcal{F}$ is a family of bargaining solutions satisfying Assumption 3. Then $\mathcal{F}$ is single-period identified if and only if $\mathcal{F}$ is the family of Kalai proportional bargaining solutions.*

The proof is on Page 17 and rests on several distinct claims, some of which may be of independent interest.

An important condition is *concavity*. Myerson (1981) defines concavity for more general social choice problems which lack a notion of disagreement. However, bargaining solutions are essentially special cases of social choice functions applied to a subset of social choice problems, so the concept can be extended to bargaining games.

Definition 5 (Concavity)**.** *For bargaining games $G = (S, v^D)$ and $G' = (T, u^D)$ and $\lambda \in [0, 1]$, define $\lambda G + (1-\lambda)G' = (\lambda S + (1-\lambda)T, \lambda v^D + (1-\lambda)u^D)$. A bargaining solution f satisfies concavity if for all G, G' such that $\{G, G'\}$ satisfies Assumption 1 and all $\lambda \in [0, 1]$, $f(\lambda G + (1-\lambda)G') \geq \lambda f(G) + (1-\lambda)f(G')$.*

The concavity property is that before the realization of uncertainty, ex ante negotiations are weakly better than ex post negotiations for all players.

Lemma 2 (Single-period identification implies concavity)**.** *Suppose $\mathcal{F}$ is a family of bargaining solutions satisfying Assumption 3, and such that there is an $f \in \mathcal{F}$ that is not concave. Then $\mathcal{F}$ is single-period unidentified.*

Proof. At a high level, suppose there is a game in which player two strictly prefers ex post negotiation with expected value $\lambda f(G) + (1-\lambda)f(G')$ to ex ante negotiation with expected value $f(\lambda G + (1-\lambda)G')$. By WPO, player one must prefer the ex ante outcome. Then the ex post outcome under f is observationally equivalent to an ex ante outcome under a solution f' that is more favorable to player two. For details, see Appendix A.2. □

Concavity is unique to two classes of bargaining solutions: utilitarian bargaining and proportional bargaining.

Lemma 3 (Concavity and IIA implies utilitarian or egalitarian)**.** *Suppose f satisfies WPO, IIA, and concavity. Then either (i) $f \Leftrightarrow f_\tau^{\text{Kalai}}$ for some $\tau \in [0,1]$, or (ii) f is utilitarian in the sense that there is a $\tau \in [0,1]$ such that $f(S, v^D) \in \operatorname{argmax}_{s \in S: s \geq v^D} \tau(s_1 - v_1^D) + (1 - \tau)(s_2 - v_2^D)$ for all S, v^D.*

Proof. The claim is a gentle extension of Myerson (1981)'s Theorem 2, but requires care to handle disagreement constraints. The details are in Appendix A.2. □

Thomson (1994) provides more background on utilitarian bargaining. A simple property is that it cannot identify bargaining weights in single-period games.

Lemma 4 (Single-period identification implies not utilitarian)**.** *Suppose for all $t \in [0,1]$, $f_t^{(Util)}(S, v^D) \in \operatorname{argmax}_{s \in S: s \geq v^D} t(s_1 - v_1^D) + (1 - t)(s_2 - v_2^D)$ for all S, v^D. Then if $f_\tau^{(Util)}$, $f_{\tau'}^{(Util)}$ for some $\tau \in (0,1)$, $\tau' \in [0,1]/\tau$, then $\mathcal{F}$ is single-period unidentified.*

Proof. Without loss of generality assume that $\tau, \tau' < 1$. Write $f = f_\tau^{(Util)}$ and $f' = f_{\tau'}^{(Util)}$. Consider the games b and b' with $EA = 0$ and f and f', respectively, applied to $u_1(p \mid R) = 1 - p$ and $u_2(p \mid R) = \frac{\min\{\tau,\tau'\}/2}{1-\min\{\tau,\tau'\}/2} p$. Then $p^* = 0$ under both games, so $b \Leftrightarrow b'$. Now let b'', b''' be f and f', respectively, applied to $u_1(p \mid R) = 1 - p$ and $u_2(p \mid R) = \frac{(\tau+\tau')/2}{1-(\tau+\tau')/2} p$. Then $b'' \not\Leftrightarrow b'''$, so $f \not\Leftrightarrow f'$ and $\mathcal{F}$ is single-period unidentified. □

I am almost ready to prove Theorem 1. I will make use of a moment for identifying Kalai bargaining weights regardless of the value of EA.

Lemma 5 (Single-period Kalai implies expected transfers)**.** *Consider a game $(e, j, f_\tau^{\text{Kalai}})$ with outcome p_e^*. Then $\tau E[u_2(p_e^* \mid R)] = (1 - \tau) E[u_1(p_e^* \mid R)]$.*

Proof. For games with $EA = 1$, the claim is immediate. For games with $EA = 0$, the realized price satisfies $\tau u_2(p_e^* \mid R) = (1 - \tau) u_1(p_e^* \mid R)$ almost surely, so that the expected transfer holds by iterated expectations. □

Proof of Theorem 1. Direction 1: if $\mathcal{F}$ is single-period identified, then it is the family of Kalai proportional solutions. Proof by contradiction. By Lemma 2, if $\mathcal{F}$ is single-period identified, then each element is concave. By Lemma 3, this and IIA implies that each element is utilitarian or proportional. By Lemma 4, $\mathcal{F}$ contains at most two bargaining solutions that are not proportional, and both are utilitarian. Then there is an $m > 0$ such that the game $EA = 0$, $u_1(p) = mp$, $u_2(p) = 1 - p$ has $p^* \in \{0, 1\}$ for all utilitarian solutions in $\mathcal{F}$. But by the span assumption applied to this game, for all $\tau \in (0, 1)$, $f_\tau^{(Kalai)} \in \mathcal{F}$. Now suppose $f \in \mathcal{F}$ is utilitarian and not proportional. By single-period identification and $f_\tau^{(Kalai)} \in \mathcal{F}$ for all interior τ, f must predict gain-from-trade shares of either one or zero always. But then f is also proportional. Therefore $\mathcal{F}$ only includes proportional solutions. By applying the span assumption to the game $u_1(p) = p$, $u_2(p) = 1 - p$, $\mathcal{F}$ is the family of proportional solutions.

Direction 2: if every $f \in \mathcal{F}$ is proportional, then f is single-period identified. Let data be generated as in Section 2.3 from some $f \in \mathcal{F}$ with bargaining weight τ. Let $\hat{\tau}$ solve $(1 - \hat{\tau})E[u_1(p^* \mid R)] = \hat{\tau}E[u_2(p^* \mid R)]$. By WPO and the existence of a p generating strict gains from trade, at least one of $E[u_1(p^* \mid R)]$ and $E[u_2(p^* \mid R)]$ must be strictly positive. Therefore $\hat{\tau}$ is unique. By Lemma 5, $\hat{\tau} = \tau$. It remains to show that no other bargaining solution in $\mathcal{F}$ is observationally equivalent. Suppose $f' \in \mathcal{F}/f$. Then by Lemma 5, f' generates a different moment. Therefore $f \not\Leftrightarrow f'$. □

The remainder of this work focuses on the Kalai proportional solution. I now proceed to a world in which contracts remain in place for multiple periods.

3 Bilateral Bargaining in a Multiperiod World

I characterize nonidentification and identification in a setting with two agents negotiating with transferable utility in discrete time. I first show that the Kalai model is not identified with multiperiod agreements. I then describe a static utility property called step-by-step, which I show simplifies recursive Kalai proportional bargaining and allows me to provide sufficient conditions for identification.

3.1 The Multiperiod Kalai Model is Only Identified with Single-Period Agreements

Consider the following family of infinitely-lived games with transferable utility. Period t is as follows:

1. Players 1 and 2 learn their period utility shocks $\varepsilon_t = (\varepsilon_{t1}, \varepsilon_{2t})$, observable state x_t in a discrete space $\mathcal{X}$, unobservable information i_t, and inflation rate $\phi_t > 0$. I write $h_t = (\varepsilon_t, x_t, i_t, \phi_t)$.

2. If the last period ended with a contract in place ($e_{t-1} = 0$ and $\ell_{t-1} > 1$), then the transfer is $p_t = \phi_t p_{t-1}$, the remaining length is $\ell_t = \ell_{t-1} - 1$, the expiration is $e_t = 1\{\ell_t \leq 1\}$, and I write $R_t = 0$ for no new negotiation.

3. If last period ended with an expiring or expired contract ($e_{t-1} = 1$), then the players can bargain by mutual assent and choose a starting transfer p_t for a T-period agreement (so that $\ell_t = T$, $R_t = 1$, and the expiration is $e_t = 1\{\ell_t \leq 1\}$) or can disagree (so that $p_t = 0$, $\ell_t = 0$, $e_t = 0$, and $R_t = 0$).

4. *Utility and transition.* Player i gets flow utility $1\left\{\ell_t > 0\right\}\left(u_i\left(x_{i,t}\right) + \varepsilon_i + p_t(2i-3) - r_i R_t\right),$ where r_i is player i's new-contract negotiating cost. h_{t+1} is drawn from some fixed distribution $P(h' \mid h)$. The econometrician observes (p, ℓ, x).

I implicitly impose that h follows a Markov process. I allow a negotiation cost to validate successful agreements; I discuss this further in Section 4.1.3.

I impose TU to simplify the analysis of nonparametric identification. As a result, these games are not exactly a generalization of the previous section's games, and the identification results here also extend to recursive TU Nash. If I considered more general games in which utility could be nontransferable, then the Nash nonidentification of the previous section would carry over.

I proceed assuming the agents satisfy certain regularity conditions.

Assumption 4 (Magnac and Thesmar (2002))**.** $E[\varepsilon_{t,i} \mid x] = 0$, the agents have perfect expectations about the law of motion $P(h' \mid h)$, and $P(\varepsilon', x', \phi', i' \mid h) = P(\varepsilon' \mid x, \phi)P(x', \phi', i' \mid x, \phi, i)$.

These are essentially the regularity conditions maintained by Magnac and Thesmar, but without imposing absolute continuity of ε and including the possibility of unobserved information about future states. The final assumption is a conditional independence assumption that there are no persistent unobservable drivers of utility.

For simplicity, I follow Collard-Wexler et al. (2019) and focus on games in which agreements are always formed. Selection of disagreement on unobservables introduces a selection bias that can be corrected with distributional knowledge, but which I leave for future work. Further, even in the case of $T = 1$ and no unobserved utility, it is impossible to separately identify the levels of $u_i(x)$ from r_i: one can increase u_i and r_i to achieve the same real outcome. With T-period agreements, it turns out that one can only hope to identify

$$\tilde{u}_i(x) \equiv u_i(x) - \frac{r_i}{\sum_{t=1}^{T} \beta^{t-1}}. \tag{1}$$

Further, even in the case of no unobserved utility, the payment data cannot separately identify the components of

$$\tilde{p}(x) \equiv -\tau \tilde{u}_1(x) + (1 - \tau)\tilde{u}_2(x). \tag{2}$$

I implicitly assume that the researcher has access to instruments that identify bargaining weights from $\tilde{p}$. In Section 2, the instruments are the known realized utility functions.

A structure b is a set b of current-period effective utility functions $\tilde{u}_i(x)$, next-period value functions $v(p,h,\ell)$ for $0 \le \ell' \le T$, information i, discounting rate $\beta \in (0,1)$, and new-contract choice functions $d^*(h) = (\ell^*(h), p^*(h))^T$ such that if $\ell > 0$,

$$\begin{aligned}
v_i(p,h,\ell) &= u_i(x) + \varepsilon_i + p + \beta E[v(\phi' p', h', \ell - 1) \mid h], \\
v(p_0,h,0) &= (v\,(p^*(h), h, \ell^*(h)) - r)\, 1\,\{\ell^*(h) = T\} \\
v(p_0,h,0) &\ge v\,(p^*(h), h', \ell^*(h)) - r, \\
v(p_0,h,0) &\ge \beta E[v(p_0, h', 0) \mid h].
\end{aligned}$$

A structure b is a *recursive Kalai* solution with bargaining weight τ if $v\,(p^*(h), h, \ell^*(h)) - r$ always solves the Kalai proportional bargaining problem over $v(p,x,\varepsilon,T) - r$ with disagreement point $\beta E[v(p,h',0) \mid h]$ and player-one bargaining weight τ.

Definition 6 (Multiperiod identification)**.** *I write that two structures b, b' are* observationally equivalent *if $P(p', \ell', x', \phi' \mid p, \ell, x, \phi)$ are the same under b and b', in which case I write $b \Leftrightarrow b'$. For a family of structures B, I write that* traditional bargaining parameters are identified *under B if $b \Leftrightarrow b'$ implies that $\tilde{p}(x)$ are the same functions under b and b'. If traditional bargaining parameters are not identified, I write that B is* unidentified. *I write that* β is identified *under B if $b \Leftrightarrow b'$ implies that β is the same under b and b'. If traditional bargaining parameters are identified under B, I write that* all bargaining parameters are identified *under B if β is also identified and I write that* only β is not identified *under B if β is not identified.*

With single-period Kalai proportional agreements, traditional bargaining parameters are identified.

Proposition 2 (Identification of traditional bargaining parameters with single-period contracts)**.** *Let $B^{(1)}$ be the set of structures generated by recursive Kalai bargaining satisfying Assumption 4 with $T = 1$, and such that $\ell^*(h) = 1$ for all (h) in the support of the game. Then only β is not identified under $B^{(1)}$.*

Proof. First, I show that traditional bargaining parameters are identified. By Assumption 4, agreement and disagreement have the same next-period expected value. For any given τ, agreements p_t^* must satisfy

$$
\begin{aligned}
0 &= -\tau\left(u_2(x) + \varepsilon_2 + p_t - r_2\right) + (1-\tau)u\left(\tilde{u}_1(x) + \varepsilon_1 - p_t^* - r_1\right) \\
&= \tilde{p}(x) - \tau\varepsilon_2 + (1-\tau)\varepsilon_1 - p_t^* \\
E[p_t^* \mid x] &= \tilde{p}(x) + E\left[-\tau\varepsilon_2 + (1-\tau)\varepsilon_1 \mid x\right] = \tilde{p}(x),
\end{aligned}
$$

with the final line following by Assumption 4. Therefore traditional bargaining parameters are identified.

Next, I show that β is not identified. Take $u_1(x) = u_2(x) = 1$, $r = 1/2$, and $\varepsilon = 0$ constant. Let b be the Kalai proportional solution with $\tau = 1/2$ and $\beta = 0$ and let b' be the solution with $\tau = 1/2$ and $\beta = 1/2$. By inspection, $b \Leftrightarrow b'$. Therefore β is not identified. □

With multiperiod agreements, even traditional bargaining parameters are not identified.

Proposition 3 (Nonidentification and static representation with multiperiod contracts)**.** *Suppose $T \geq 2$, let $B^{(T)}$ be the set of structures generated by recursive Kalai models satisfying Assumption 4 with contract length T and such that $\ell^*(h) = 1$ for all (h) in the support of the game. Then $B^{(T)}$ is unidentified.*

Proof. Let data follow from some structure b with $\beta > 0$ and $\varepsilon = 0$ and no uncertainty, so that d^* is deterministic conditional on x. Let b' be a structure with $\beta = 0$, $\tilde{p}(x) = E[v(p, h, T) \mid x]$, and $\varepsilon = 0$. Then $b \Leftrightarrow b'$, so that b and b' are observationally equivalent. □

I propose sufficient conditions for identification. My proofs will depend on an essential tool for dynamic recursive bargaining problems: the step-by-step property for static games.

3.2 The Step-by-Step Property for Recursive Bargaining

For illustration, consider the Kalai proportional bargaining problem with two-period agreements ($T = 2$), no unobservable utility ($\varepsilon = 0$), and constant inflation ($\phi_t = 0$). To avoid division by zero, assume $\tau \in (0, 1)$. Then the sides choose a price $p^*(x)$ to satisfy:

$$\frac{\tau}{1-\tau} = \frac{v_1\left(p^*(x), x, \varepsilon = 0, \phi = 1, \ell = 2\right) - \beta E\left[v_1(p^*(x'), x', 0, 1, 2) \mid x\right]}{v_2\left(p^*(x), x, \varepsilon = 0, \phi = 1, \ell = 2\right) - \beta E\left[v_2(p^*(x'), x', 0, 1, 2) \mid x\right]}. \tag{3}$$

This is a recursive bargaining problem.

The solution turns out to be the same if the negotiators replace the value of one-period disagreement with the value of two-period disagreement. Hypothetically, imagine the sides bargain over a two-period-agreement price p^{**} subject to disagreeing for two periods and then returning to equilibrium. This disagreement point is implausible but useful: under two-period commitment, the associated function $p^{**}(x)$ would satisfy:

$$\frac{\tau}{1-\tau} = \frac{v_1\left(p^{**}(x), x, 0, 1, 2\right) - \beta^2 E\left[v_1(p^*(x''), x'', 0, 1, 2) \mid x\right]}{v_2\left(p^{**}(x), x, 0, 1, 2\right) - \beta^2 E\left[v_2(p^*(x''), x'', 0, 1, 2) \mid x\right]}. \tag{4}$$

The two-period-disagreement problem is much easier to solve, because $v_i(p, x, 0, 1, 2) = u_i(x) + \beta E[u_i(x') \mid x] + (1 + \beta)(2i - 3)p + \beta^2 E\left[v_i(p^*(x''), x'', 0, 1, 2) \mid x\right]$, yielding the finite-horizon negotiation problem:

$$\frac{\tau}{1-\tau} = \frac{u_1(x) + \beta E[u_1(x') \mid x] - (1 + \beta)p^*(x)}{u_2(x) + \beta E[u_2(x') \mid x] + (1 + \beta)p^*(x)},$$

which is the price the sides would negotiate if they formed a two-period agreement relative to two-period disagreement in a two-period world. Beginning in the third period, outcomes are the same under the agreement and hypothetical disagreement.

The equivalence of p^* and p^{**} is as follows. Write $G_i(x)$ for the (possibly random) gains from trade achieved by negotiation at state s. By Equation (3), $G_1(x) = \frac{\tau}{1-\tau} G_2(x)$ for all x.

Then, by expanding Equation (3) and applying the law of iterated expectations,

$$\frac{\tau}{1-\tau} = \frac{v_1\left(p^*(x), x, 0, 1, 2\right) + \beta \frac{\tau}{1-\tau} E\left[G_2(x) \mid x\right] - \beta^2 E\left[v_1(p^*(x''), x'', 0, 1, 2) \mid x\right]}{v_2\left(p^*(x), x, 0, 1, 2\right) + \beta E\left[G_2(x) \mid x\right] - \beta^2 E\left[v_2(p^*(x''), x'', 0, 1, 2) \mid x\right]}.$$

By simple algebra, this constraint is the same as if $G_2(x) = 0$ deterministically, which yields the two-period-disagreement problem Equation (4).

This is the step-by-step property in action. The step-by-step property is that the outcome of bargaining in one shot is the same outcome as reaching a first-step agreement, updating the disagreement point to the first-step agreement value, and then bargaining over any remaining surplus. Imagine negotiating the price of a new car by first negotiating over the price, holding any upgrades fixed; and then negotiating over price and upgrades, relative to the outcome of the constrained bargain. The step-by-step property ensures that this process will generate the same agreement as negotiating over the full surplus at once. Mathematically, if $v^D \in T \subseteq S$, then $f(S, v^D) = f(S, f(T, v^D))$. Kalai (1977) and Roth (1979) show that this property is essentially unique to Kalai proportional bargaining, although it is difficult to state precisely in the real world.

The step-by-step property is useful for simplifying recursive bargaining problems. In many dynamic settings, a pair bargain relative to the value of agreeing next period under a new bargaining state. This generates a complicated recursively defined bargaining problem. The one-disagreement bargain can be viewed as a first-step agreement relative to the value of agreeing in two periods. Applying the step-by-step property to the gains generated by disagreeing for one period rather than two periods, the value of one disagreement can be replaced by two, three, or infinitely-lived disagreement without changing the equilibrium agreement. The resulting simplification is qualitatively similar to finite dependence in dynamic discrete choice (DDC) models (Arcidiacono and Miller, 2011).

Figure 2 illustrates the step-by-step property. Recall the intuition of the Kalai proportional solution as choosing the best agreement along a surplus-splitting ray. In Figure 2(b),

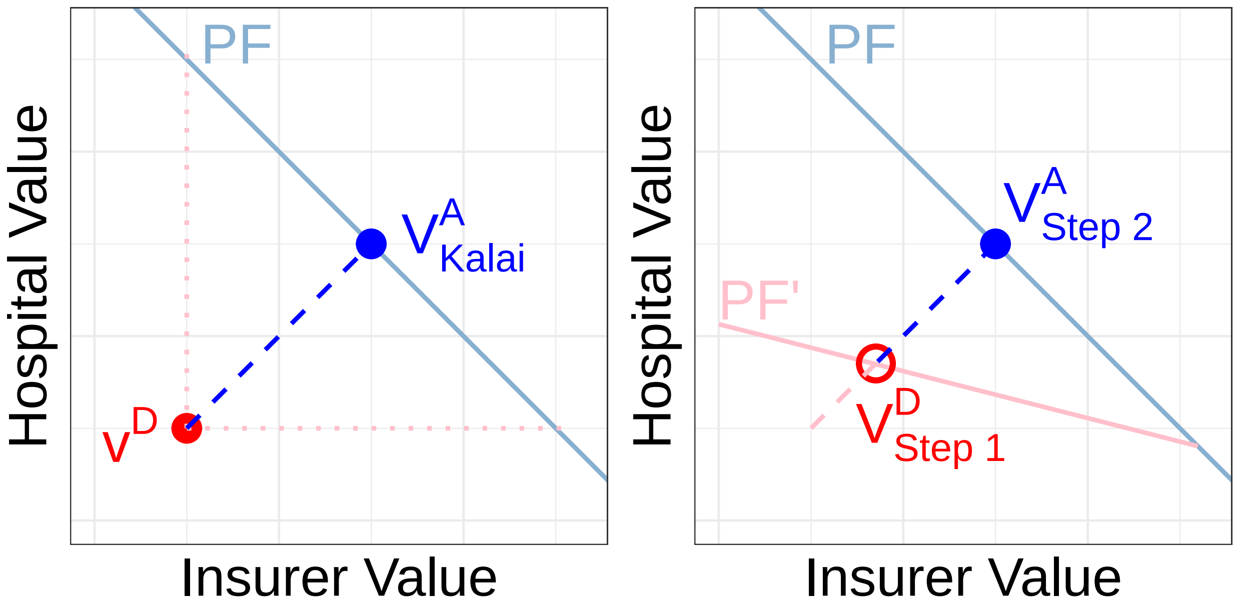

Figure 2: Left: Recall the Kalai proportional bargaining solution for choosing an agreement V^A_{Kalai} (blue) on a Pareto frontier PF relative to the value of disagreement v^D (red), with proportional split illustrated by dashed blue line. Right: illustrating the step-by-step property for a final agreement $V^A_{\text{Step 2}}$ (blue) recursively defined through bargaining relative to a first-step agreement $V^D_{\text{Step 1}}$ (red), chosen through applying the bargaining solution to negotiations over some smaller Pareto frontier PF' (pink, with dashed line indicating gains) relative to the full disagreement point v^D (omitted).

instead of negotiating over the full agreement, the sides first negotiate over a smaller Pareto frontier (pink) to reach a first-step agreement. The first-step agreement then becomes the disagreement point in the second-step bargain over the full surplus, leading to a final agreement. This generates the same outcome as bargaining over the full surplus Figure 2(a), because the first-step agreement corresponds to moving the disagreement value upwards along the same surplus-splitting ray.

I now leverage this property to replace the value of one-period disagreement on a T-period contract with the value of T-period disagreement. This allows me to prove sufficient identification conditions for recursive Kalai bargaining with multiperiod agreements over uncertain and NTU Pareto frontiers.

3.3 Identification of Kalai with Multiperiod Agreements

I provide two sufficient conditions for identification.

The first sufficient condition for identification is based on a remark by Rust (1994), who writes that in dynamic discrete choice models with no observed components of utility, some partial identification may come from "agents who make different choices in the same state." In recursive Kalai bargaining games, I show that this condition permits nonparametric identification.

Proposition 4 (Identification without unobservables)**.** *Let $B^{(T,1)}$ be the set of $b \in B^{(T)}$ such that there are no unobserved components of utility ($\varepsilon = 0$) and the number of (p, x) combinations observed is more than the number of values of x observed. Then under appropriate rank conditions, all bargaining parameters are identified under $B^{(T,1)}$.*

Proof. See Appendix A.2 □

This logic may enable identification in the presence of unobservables under an i.i.d. restriction on the distribution of ε that I do not pursue.

The second sufficient condition is motivated by empirical practice, which tends to favor GMM estimation. I show that an instrument satisfying an exclusion restriction can suffice to identify the discounting rate.

Proposition 5 (Identification from inflation expectations)**.** *Suppose z_t is a real-valued observed coarsening of i_t satisfying an exclusion restriction $\{x_t, \varepsilon_t\}_{t=t_0}^{t_0+T-1} \perp\!\!\!\perp z_{t_0} \mid x_{t_0}, \phi_{t_0}$, and satisfying a relevance condition that for all $\delta > 0$, $E\left[\prod_{t_0 < v \leq t_0+\delta} \phi_v \mid x_{t_0}, \phi_{t_0}, z_{t_0}\right]$ is strictly increasing in z_{t_0}. Suppose further that there is an x_{t_0}, ϕ_{t_0} satisfying a non-zero condition $E\left[p^*_{t_0} \mid x_{t_0}, z_{t_0}, \{\phi_t\}_{t=t_0}^{t_0+T-1}\right] > 0$ for all $z_{t_0}, \{\phi_t\}_{t=t_0}^{t_0+T-1}$. Then under appropriate rank conditions, all bargaining parameters are identified under $B^{(T)}$.*

Proof. See Appendix A.2. □

I view this second condition as more plausible in empirical use. The key restriction is that z_t affects bargaining only through expectations about inflation, and there is a clear direction of prices so that inflation expectations affect starting prices. Empirical practice also can call for interactions across negotiations by different players, which I add in the next section.

4 Multilateral Bargaining in a Multiperiod World

Now imagine that many agents negotiate staggered and interacting contracts in a changing and uncertain world. This will call for a model in which in every subgame, strategies form a Nash equilibrium of the game defined by later behavior, and the contract formed by a given pair always follows from applying a pair-specific bargaining solution to expected value functions at the current state of the world. The arguments above in more restricted two-agent games argue for applying recursive Kalai proportional bargaining. I call the associated models *Nash-in-Kalai* models.

4.1 Model Timing

I study dynamic markets of the following form.

Time is indexed by t. One unit of time is divided into $m \geq 1$ periods, with the time of period s written as t_s: $t_1 = 0$, $t_2 = 1/m$, $t_{1+m} = 1$, and so on. The per-period discounting rate is equal to $\beta^{1/m} \in [0, 1)$, with myopia corresponding to $\beta = 0$.

The timing in each period t_s is as follows:

1. *Information is revealed and non-bargaining choices are made.* Information $\mathcal{I}_{t_s}$ is revealed, possibly through multiple sub-periods. The resulting information includes the history of the game, period pre-transfer utility functions $u_{t_s i}(\mathbb{C}_{t_s})$, period demand functions $D_{t_s,ij}(\mathbb{C}_{t_s})$, null contracts $\mathbb{C}_{0,ijt_s}$, the set of feasible bilateral contracting states $\mathcal{C}_{ijt_s}$ (either a single renewal contract or a set including a null contract), and any information about future values. Every feasible contract ij includes p_{ijt_s}, the net i-to-j

transfer per unit of equilibrium demand $D_{t_s,ij}(\mathbb{C}_{t_s})$ provided in period t_s.

2. *Contracts are bargained.* For $i < j$ pairs with more than one feasible contract, new contracts $\hat{\mathbb{C}}_{ijt_s}$ are chosen through bilateral Kalai proportional bargaining relative to the null contract, with j receiving bargaining weight $\tau_{ij} \in [0,1]$. I write the number of new contracts formed by firm i as $\hat{R}_{it_s}$.

3. *Flow profits are formed.* Flow profits for agent i at contract state $\hat{\mathbb{C}}_{t_s}$ with associated j-to-i prices $\hat{p}_{ijt_s} = -\hat{p}_{jit_s}$ and network of firms with agreements $\mathcal{G}_{it_s}$ are equal to

$$u_{t_s i}(\hat{\mathbb{C}}_{t_s}) + \sum_{j \in \mathcal{G}_{it_s}} \hat{p}_{ijt_s} D_{t_s,ij}(\hat{\mathbb{C}}_{t_s}) - r_i \hat{R}_{it_s},$$

where $r_i \geq 0$ is the cost of validating a new contract.

Definition 7 (Nash-in-Kalai equilibrium)**.** *A* Nash-in-Kalai equilibrium *is a conditional random variable distribution* $\mathcal{I}_{t_s} \mid (\mathcal{I}_{t_{s-1}}, \mathbb{C}_{t-1})$*, a bilateral contract choice distribution* $\hat{\mathbb{C}}_{t_s,ij}(\mathcal{I}_{t_s})$*, and recursive value functions* $V_i(\mathcal{I})^{(s)}$ *for various stages* s *such that (i) the value function constraints in Appendix A.1 hold, (ii) all strategies are Markov strategies, (iii) any strategies involved in the formation of* $\mathcal{I}_{t_s}$ *in Stage 1 satisfy some appropriate concept of equilibrium (typically Nash equilibrium), and (iv) for every* ij *and every subgame, the expected value of* $\hat{\mathbb{C}}_{t_s,ij}(\mathcal{I}_{t_s}) \mid \mathcal{I}_{t_s}$ *when taking other strategies as given solves the Kalai proportional bargaining problem over value functions with bargaining weight* $\tau_{ij} \in [0,1]$ *on player* $j > i$*.*

I call this the Nash-in-Kalai model because when the restrictions on strategies in Stage 1 include Nash equilibrium and the bargaining problem formed in Stage 2 satisfies Assumption 1, then WPO ensures that any equilibrium forms a Nash equilibrium in recursively defined Kalai proportional bargains. Due to the bilateral nature of contracting, I have not found the recursive value functions to be useful beyond collecting terms. I present them in Appendix A.1 for the interested reader.

I discuss each stage in further detail.

4.1.1 Information is Revealed

The information updating Stage 1 is written in a generic way in order to accommodate many types of markets.

Stages 1 and 3 can be modified to include separate games. The first stage can include price-setting, contract renewal, or other strategic phases. The final stage can include demand reallocation in response to upstream prices. I focus on the bargaining stage and abstract from these strategic responses here. One could just as easily imagine strategic interactions after bargaining rather than before bargaining. Such a change could be accommodated at the cost of additional notation to track the outcome of the end-of-period response. I will eventually imagine a period to be arbitrarily short, so that a single period is almost irrelevant. A more substantive change would be if non-bargaining competition occurred at the same time as bargaining in Stage 2. Such simultaneous competition would be insubstantial if it occurred at discrete moments, but the step-by-step property would be less useful if strategic responses are revisited at every moment in which a pair may attempt to negotiate.

4.1.2 Contracts are Bargained

The core of my analysis is the bargaining Stage 2.

I assume that a contract must have a price per unit of demand, which corresponds to transfers in the scale of utility. Transferable utility corresponds to constant demand of one unit. A contract can have other characteristics: a fixed end date or an auto-renew clause; a benchmark rule for updating prices while the contract remains in place; or per-period transfers as a function of each firm's network of realized agreements (Olssen and Demirer, 2022; Ho and Lee, 2024). However, there is a meaningful restriction that flow profits depend only on utility functions that are known at the start of the period, negotiation costs, realized contracts, and predictable transfers.

I also assume that a pair chooses a contract taking as given both future strategies and all strategies for other simultaneous actions. The assumption on future strategies is to my knowledge innocuous. The restriction on simultaneous strategies is the standard passive beliefs assumption that if i or j defects and changes their behavior, neither party adjusts their behavior in, or expectations about, other strategic decisions taken at the same time (Lee et al., 2021). This decision is relatively innocuous in a dynamic market: I will have in mind a model where the length of a period tends to zero, so that simultaneous bargains are rare and typically have disagreement constraints implying an increasingly-small set.

4.1.3 Flow Profits are Formed

The representation of profits in Stage 3 as the sum of a pre-transfer flow profit function and negotiated transfers generalizes many, but not all, conceivable dynamic markets.

I implicitly rule out transfers under null contracts. The Nash-in-Kalai model could be extended to include disagreement transfers, for example through out-of-contract purchasing (Prager and Tilipman, 2022), at the cost of yet more notation.

Note that Kalai proportional bargaining with extreme weights may not be defined if the implied game fails comprehensiveness. For example, in traditional models of double marginalization with negotiation of upstream supply before a retailer sets downstream prices, the supplier's preferred price may generate positive gains from trade for a downstream retailer. In this case,the associated static game fails comprehensiveness. A corollary is that sequential-pricing double marginalization models may not admit identification in settings with possible uncertainty, and at a minimum will be complex to express.

I model a negotiation cost borne after bargaining succeeds. Real negotiation costs are borne both at the stage of preparing for negotiations (Gooch, 2019; ECG, 2020; Fletcher, 2020; Beier, 2020) and at the stage of carefully checking the terms of a potential agreement (STD TAC and Moss, 2014; PMMC, 2019; Fletcher, 2020). I model the bargaining friction as only the ex post cost to validate a potential agreement. Some work includes a sunk negotiation cost (Prager and Tilipman, 2022). Sunk costs can prevent firms from forming Pareto-efficient contracts. In a static model, sunk costs have the advantage of not entering into payments. In a forward-looking model, future sunk costs enter into current payments in a challenging way.

4.2 Expected Net Present Value Transfers Under Nash-in-Kalai Bargaining

I now show that the dynamic Nash-in-Kalai model yields a moment on expected NPV transfers that generalizes the static Nash-in-Nash moment on current transfers. The moment delivers key tractability properties. In particular, shortening the length of disagreement does not change the moment, so that a complex Nash-in-Kalai model defined in continuous time can have the exact same predictions as a particular and more tractable model defined in discrete time. The existence of a transfer moment and a tractable continuous-time foundation for a discrete-time model are key advantages for empirical work.

I maintain some regularity conditions.

Assumption 5. (Regularity conditions)
Players are risk-neutral, share rational expectations, and follow Markov strategies. If ij reach their null contract in t_s and do not negotiate a new contract in t_{s+1} then ij reach their null contract in t_{s+1}. There is a uniform transversality condition: if $\mathcal{F}_{t_r|t_s}(\mathcal{I}_{t_s})$ is the set of feasible information sets in period $t_r \geq t_s$ after information $\mathcal{I}_{t_s}$ is reached in period t_s, then

$$\lim_{h\to\infty} \sup_{\mathcal{I}_{t_s}} \sup_{\mathcal{I}_{t_{s+h}} \in \mathcal{F}_{t_{s+h}|t_s}(\mathcal{I}_{t_s})} \sup_i \beta^{h/m} \left| V_i^{(1)}(\mathcal{I})(\mathcal{I}_{t_{s+h}}) \right| = 0.$$

By repeated application of the step-by-step property, the econometrician (and negotiators) can replace the value of disagreement with the value of impasse. In the ij impasse point, i and j continually attempt to reach an agreement, but surprise all actors and continue to disagree without abandoning negotiations. Under a fixed set of Markov strategies, the impasse point is uniquely-defined regardless of whether disagreement is unilateral or bilateral. The impasse point captures the dynamic intuition of static Nash-in-Nash disagreement: everyone behaves anticipating ij to reach an agreement, but ij always surprise the market and disagree.

The impasse point is typically defined in discrete time, even when bargaining is conducted in continuous time. In particular, the value of impasse only depends on ij bargaining states through states at which others respond to an anticipated ij agreement. Most ij bargaining states under impasse have no impact on the value of impasse, and so can be ignored for the purposes of calculating the Nash-in-Kalai solution.

The dynamic Nash-in-Kalai bargaining model yields a moment on expected NPV transfers, and as a result the econometrician can construct moments on transfers for estimation. The moment naturally generalizes the static Nash-in-Nash bargaining moment to incorporate multiple periods of gains from trade.

Theorem 2 (Nash-in-Kalai Moment)**.** *Consider a dynamic Nash-in-Kalai equilibrium that satisfies Assumption 5. Suppose players $i < j$ form a contract in a subgame time t_0 that remains in place through the (potentially random) terminal time t^* with (potentially random) realized prices p^*_{ijt}. Then the expected NPV of realized transfers $D_{t_s,ij}p^*_{ijt_s}$ at the moment of contract formation is equal to the sum of the expected NPV of Nash-in-Nash flow transfers, a negotiation cost transfer, and an impasse repricing transfer term:*

$$\mathbb{E}_{t_0}\left[\sum_{t_0 \le t_s \le t^*} \beta^{\frac{t_s - t_0}{m}} D_{t_s,ij} p^*_{ijt_s}\right] = \text{Pay}_{NiN} + \text{Pay}_{NC} + \text{Pay}_{IRT}, \tag{5}$$

where the expected NPV of static Nash-in-Nash transfers ($\tilde{p}$ in Equation (2)) is:

$$\text{Pay}_{NiN} = \mathbb{E}_{t_0}\left[\sum_{t_0 \le t_s \le t^*} \beta^{\frac{t_s - t_0}{m}} \begin{pmatrix} -\tau_{ij} & ([\Delta_{ij} u_{it_s}] + [\Delta_{ij} T_{it_s,-j}]) \\ +(1-\tau_{ij}) & ([\Delta_{ij} u_{jt_s}] + [\Delta_{ij} T_{jt_s,-i}]) \end{pmatrix}\right], \tag{6}$$

the negotiation cost transfer Pay_{NC} *is equal to* $\tau_{ij} r_i - (1-\tau_{ij}) r_j$*, and the impasse repricing* *transfer* Pay_{IRT} *is:*

$$\text{Pay}_{IRT} = E_{t_0}\left[\sum_{t_s \geq t_0} \beta^{\frac{t_s - t_0}{m}} \left\{ \begin{array}{cc} -\tau_{ij} & \left(\hat{\pi}^A_{it_s} + \hat{T}^A_{it_s} - \hat{\pi}^D_{it_s} - \hat{T}^D_{it_s}\right) \\ +(1-\tau_{ij}) & \left(\hat{\pi}^A_{jt_s} + \hat{T}^A_{jt_s} - \hat{\pi}^D_{jt_s} - \hat{T}^D_{jt_s}\right) \end{array} \right\}\right], \tag{7}$$

where $\hat{\pi}^A_{t_s,k}$ *and* $\hat{T}^A_{t_s,k}$ *correspond to the (potentially random) path of profits and net transfers if* ij *enter their impasse point beginning in period* $t^* + 1$ *and the* ij *equilibrium contract is replaced with the* ij *null contract in periods* t_0 *through* t^**, and where* $\hat{\pi}^D_{t_s,k}$ *and* $\hat{T}^D_{t_s,k}$ *correspond to those paths if* ij *enter their impasse point beginning in period* t_0*.*

Proof. The claim follows by repeated application of the step-by-step property. For details, see Appendix A.2. □

In many empirical applications, the realized transfers $D_{t_s,ij}p^*_{ijt_s}$ can be observed, the flow Nash-in-Nash transfers in Pay_{NiN} depend on estimable demand functions and a small number of bargaining parameters, and Pay_{NC} depends on only a few parameters. In models like Lee and Fong (2013) with inertia, the flow Nash-in-Nash payments may be more subtle.

The term Pay_{IRT} accounts for the fact that in future periods, the Nash-in-Nash disagreement point differs from the Nash-in-Kalai impasse point. The static Nash-in-Nash gains used in Pay_{IRT} are calculated relative to disagreement under the non-ij contracts formed in equilibrium, while the Nash-in-Kalai gains are calculated relative to disagreement under the non-ij contracts formed when continually expecting expecting ij to exit impasse. Under myopia ($\beta = 0$) or single-period contracts, Pay_{IRT} is equal to zero. In the absence of negotiation costs, under myopia ($\beta = 0$) or single-period contracts, Equation (5) reduces to the TU Nash-in-Nash transfer $-\tau_{iij}\left([\Delta_{ij}u_{it_0}] + [\Delta_{ij}T_{it_0,-j}]\right) + (1-\tau_{iij})\left([\Delta_{ij}u_{jt_0}] + [\Delta_{ij}T_{jt_0,-i}]\right)$.

Theorem 2 holds whether the market is in steady state, in a stationary equilibrium that is vulnerable to change, or is nonstationary. When the market is in steady state, the predicted transfer is especially simple.

Corollary 1 (Steady state Pay_{IRT} is zero)**.** *Consider a Nash-in-Kalai equilibrium such that profit functions and contracts* $\hat{\mathbb{C}}_t$ *are constant in equilibrium, and the market in steady state in the sense that any subgame of the equilibrium outcome, if a pair ij defects and disagrees for one period, then profit functions and contracts return to the original equilibrium the next period. Then in every bargain,* $\text{Pay}_{IRT} = 0$*.*

Proof. The path of profit functions, contracts, and transfers in future periods are the same on the immediate-agreement and one-period-disagreement paths, other than future negotiation costs that will be split proportionally. Thus, $\text{Pay}_{IRT} = 0$. □

Corollary 1 is particularly useful for analyzing theoretical markets. Real-world markets are not in steady state, but so long as a market is close to steady state, Corollary 1 implies that approximating Pay_{IRT} to zero will produce an approximately valid moment. This is the strategy I follow in my empirical application.

5 Empirical Application

I apply the proposed empirical model to data on hospital–insurer bargaining from West Virginia. I prove a high-level summary here. For more detail on the data and setting, see Dorn (2025b). For more detail on the estimation procedure, see the companion paper Dorn (2025a).

The main dataset is panel information on contract formation and expiration by hospital-insurer-year from West Virginia between 2005 and 2015. I also have inpatient data on visits by residence and age for leading insurers. The model is a dynamic extension of the Ho and Lee (2017) model: hospitals agree to receive reduced payments in exchange for an insurer steering enrollees to the hospital; wider networks make insurance plans a higher-quality good when selling to consumers; and some consumers become sick and need care at a hospital. I focus on inpatient care. I assume that all sick patients receive care somewhere and always choose a provider within their insurer's network. I differentiate between a set of medium and large insurers that are modeled, and a tail of "nonmodeled" insurers.

There are known to be contracting externalities in this setting. Imagine a simple single-period world with one monopolist insurer and two hospitals. The insurer simultaneously sets premiums ϕ and negotiates with the two hospitals over a price per unit of care. After premiums are set, consumers choose insurance. Consumers all become sick, and then flip a fair coin to learn their preferred hospital. Consumers go to their preferred hospital if possible, the other hospital if impossible, and do not purchase insurance if the network is empty. When bargaining with hospital 2, the insurer knows that part of the value of the hospital is diverting patients away from hospital 1. As a result, a higher anticipated hospital-1 price leads to a higher hospital-2 price, generating contracting externalities. These contracting externalities have been modeled through anticipated simultaneous agreements, yielding the TU Ho and Lee (2017) model that I build on.

When contracts are staggered and negotiators are forward-looking, contracting externalities are internalized. Imagine that hospital 1 and hospital 2 bargain in alternating periods. When the insurer negotiates with hospital 1, they know that their negotiated price will have a positive effect on the price they negotiate with hospital 2 in the next period. These effects are asymmetric, yielding a NTU model and all of the issues of the previous sections.

Real-world contracts are meaningfully staggered. Figure 3, borrowed from the companion paper Dorn (2025a), shows that for both fixed-length contracts associated with larger insurers and auto-renew contracts associated with smaller insurers, contracts consistently remained in place for three years or longer. Many contracts remained in place for a decade or more. Different lengths imply some degree of staggering. Appendix Figure 4 shows that contracts would be staggered even within a given year. As a result, if negotiators are forward-looking, the bargaining model will be NTU.

I estimate a Nash-in-Kalai model, leveraging Theorem 2 for estimation. Dorn (2025b) and Dorn (2025a) show that hospitals and insurers had predictably different variation in contract

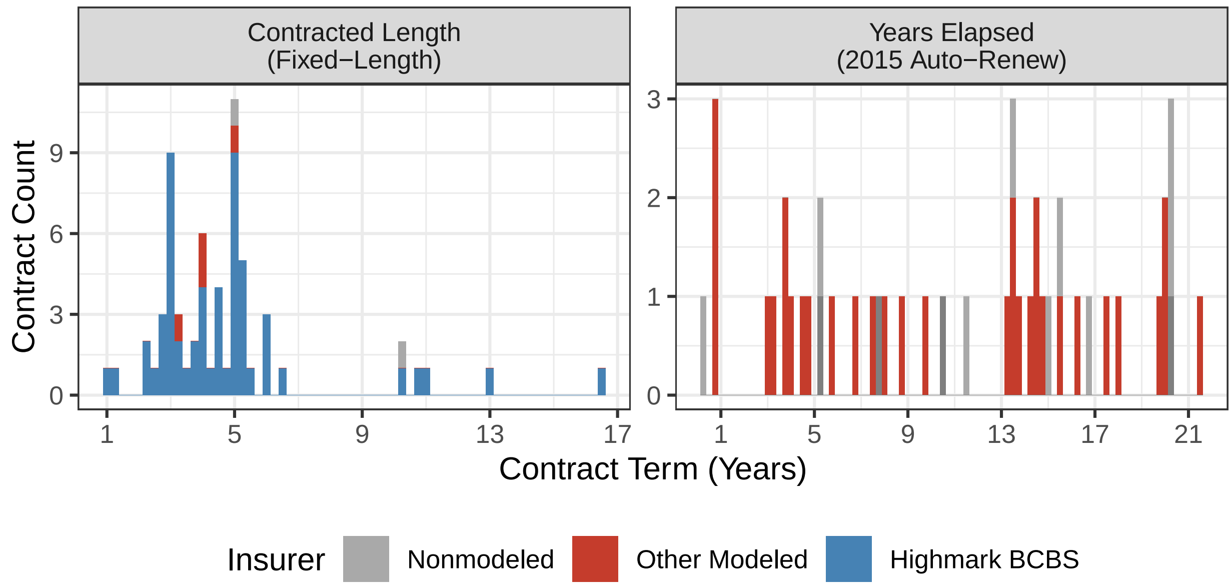

Figure 3: Contracts are multiyear. Distribution of reported contract lengths for fixed-term contracts in general (left panel) and auto-renew contracts with reported formation dates as-of fiscal 2015 (right panel). Colors indicates insurer. The two nonmodeled contracts correspond to Wheeling–Pittsburgh Steel.

lengths and payment rates, so I use insurer and hospital-size-group indicator variables as instruments based on Proposition 5. I model bargaining weights as featuring an interaction of insurer j and hospital i effects:

$$\log\left(\frac{\tau_{ij}}{1-\tau_{ij}}\right) = \delta_j + \tau^{Size}\log(Size_i),$$

where δ_j is an insurer fixed effect, τ^{Size} is a hospital-size coefficient, and $Size_i$ is a measure of hospital system size from the start of the dataset. I estimate hospital demand based on Highmark BCBS patients, who are in-network at every hospital, and estimate insurer demand as a function of network quality using institutional features of premium-setting. For more details on estimation, see the companion paper Dorn (2025a).

I estimate three bargaining models. The *Only-2015* model represents estimates based on the standard approach of taking all data in a market over a short period and estimating a Nash-in-Nash model treating all contracts as short-lived. This model is TU, so the Only-

2015 model can be interpreted as a Nash-in-Kalai model. The *Myopic* model uses the same demand estimates, but estimates bargaining parameters for new contracts in all years. By the arguments of Section 2, myopia is needed to identify a Nash-in-Nash model with staggered contracts. Estimates under myopia can also be interpreted as Nash-in-Nash or Nash-in-Kalai. Finally, the *Forward-Looking* model uses the same bargains and estimated demand functions as the Myopic model, but allows for negotiators to be forward-looking under specifically the Nash-in-Kalai model.

	Parameter				
	β	τ_{BCBS}	τ_{HPUOV}	τ_{FP}	$-\tau^{Size}$
Only-2015	·	0.487**	-7.54	0.694***	3.354
(Nash/Kalai)	(·)	(0.191)	(17.204)	(0.175)	(22.875)
Myopic	·	0.876***	0.825***	0.861***	1.037***
(Nash/Kalai)	(·)	(0.012)	(0.232)	(0.034)	(0.199)
Forward-Looking	0.899***	0.854***	0.877***	0.889***	0.989***
($\text{Pay}_{IRT} = 0$)	(0.03)	(0.006)	(0.026)	(0.005)	(0.028)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 1: Estimated bargaining and patience weights for the only-2015 (first row), myopic (second row), and preferred forward-looking (third row) bargaining models. The MCO τ_j bargaining weights represent the estimated bilateral share τ_{ij} evaluated for a hospital i with size equal to the average bargain value, and include estimated heterogeneity between Highmark BCBS (BCBS), the regional insurer HPUOV, and the modeled for-profit insurers (FP). The estimated forward-looking model overwhelmingly rejects myopia, and the only-2015 model produces implausible estimated bargaining weights. For additional parameter estimates, see Appendix Table 2.

The estimated bargaining parameters are in Table 1. The length data in Figure 3 rejects a static model like the Only-2015 model, and I find that treating old contracts as recently-formed is substantive. In this case, the only-2015 model estimates that the regional insurer HPUOV's bargaining weight is negative. This behavior is in part driven by the large estimated role of hospital size, which is associated with contract structure: if I fix $\tau^{Size} = 0$, the estimated bargaining weights for Highmark BCBS, HPUOV, and the other for-profit insurers drops to 0.365, 0.278, and 0.16, respectively. These numbers are interior, but mistakenly estimate very small bargaining weights for for-profit insurers.

Next, I turn to the Myopic and Forward-Looking models with accurate timing. These models predict limited heterogeneity in insurer bargaining weights, but important heterogeneity in hospital bargaining weights. The bargaining weights are similar whether β is allowed to be above zero or not, because demand is fairly correlated across time. Nevertheless, the forward-looking model overwhelmingly rejects the null hypothesis of myopia, estimating an (inflation-adjusted) annual discounting rate of 0.899. This parameter estimate indicates that the negotiators are likely to respond to beliefs about future conditions when choosing their starting price, indicating that there can be important dynamic incentives that the previous static literature could not model.

6 Conclusion

This paper introduces the Nash-in-Kalai solution for bargaining problems with nontransferable utility. In the presence of uncertainty over nontransferable Pareto frontiers, Nash bargaining and other scale-invariant bargaining solutions are unidentified, and only one family of bargaining solutions satisfying independence of irrelevant alternatives is identified: The Kalai proportional bargaining solution. The Kalai proportional solution's unique step-by-step property implies a valuable equivalent representation of the recursive Kalai proportional bargaining problem. I leverage this representation to provide sufficient conditions for identification of Nash-in-Kalai bargaining parameters with multiperiod agreements. The Nash-in-Kalai model admits a moment on expected NPV transfers in a wide degree of generality. As a result, the model can be applied to settings with many firms negotiating interacting agreements at different times.

My empirical analysis leverages a panel dataset that includes hospital–insurer contract timing to study one setting with staggered contract negotiations. Hospital–insurer contracting in West Virginia was meaningfully staggered, so that forward-looking bargaining is nontransferable utility. I estimate an empirical model generalizes the Ho and Lee (2017) static TU model to include NTU dynamics from staggered contract formation. I find that a static model with an inaccurate conjecture of timing would yield a misleading understanding of bargaining weights. I compare two estimators with accurate timing: a model that imposes myopia to achieve feasibility of non-Kalai models, and a Nash-in-Kalai model that allows negotiators to be forward-looking. I find that accurate timing, but not accurate time preferences, are important for accurately capturing bargaining weights. However, the estimated discounting rate is far from zero, indicating that negotiators meaningfully respond to forward-looking incentives that the previous literature could not capture.

The Nash-in-Kalai model is likely to be valuable beyond healthcare. The framework may prove useful for modeling media asset acquisition (Morrissette, 2023), programming disputes with time-bound programming (Hayes, 2023), disagreement with inertia costs (Handel, 2013), staggered team formation in the presence of aggregate wage limits (Mulholland and Jensen, 2019), and supply chain bargaining in the presence of inflation (Davidson, 1988; Baudendistel, 2023). A feasible, but non-trivial, addition for future work is to extend popular empirical search-on-the-job models (Cahuc et al., 2006; Bagger et al., 2014; Bilal et al., 2022) beyond TU bargaining. The dynamic moment of Theorem 2 provides sufficient conditions for estimation, but an important avenue for future work is characterizing plausible conditions for statistical inference in this setting. Finally, an open question for future work is whether computational tools can render a feasible Nash-in-Nash model with staggered contracts when the econometrician is willing to take a stance on information timing.

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A Additional Content

A.1 Dynamic Value Functions

I begin with the recursively-defined expected value function.

Formalizing the expected NPV profit in bargaining requires notation for the future equilibrium. Fix a period t_s in which ij have multiple feasible contracts. By assumption, the null contract $\mathbb{C}_{0,ijt_s}$ is also feasible. I will assume that strategies are Markov, so that future conditions can evolve based on the contracts realized today but not whether a null contract was reached through agreement or disagreement. Define $u_{t_r|t_s}(\mathbb{C}_{t_r} \mid \mathbb{C}_{t_s})$ to be the (potentially random) profit function that will occur in period t_r with realized agreements $\mathbb{C}_{t_s}$ this period. Similarly define $D_{t_r|t_s}(\mathbb{C}_{t_r} \mid \mathbb{C}_{t_s})$ for the demand functions. Also define $\mathcal{I}_{t_s}(\mathbb{C}_{t_s})$ to be the realized information, $\mathbb{C}_{0,k\ell t_r|t_s}(\mathbb{C}_{t_s})$ to be the realized null contracts for $k\ell$ in period t_r, $\mathcal{C}_{k\ell t_r|t_s}(\mathbb{C}_{t_s})$ for the set of feasible contracts, and $\hat{\mathbb{C}}_{t_r|t_s}(\mathbb{C}_{t_s})$ for the equilibrium contracts. Define the period t_s ij substitution function as $\left(\hat{\mathbb{C}}_{t_r,-ij|t_s}, \mathbb{C}_{t_s,ij}\right)$, which replaces the ij element of the equilibrium contract $\hat{\mathbb{C}}_{t_s}$ with $\hat{\mathbb{C}}_{t_s,-ij}$. I then extend this contract replacement to periods $t_r > t_s$ by defining $\left(\hat{\mathbb{C}}_{t_r,-ij|t_s}(\mathbb{C}_{t_s,ij}), \mathbb{C}_{t_r,ij}\right)$ to be the contract which replaces the ij element of $\hat{\mathbb{C}}_{t_r|t_s}\left(\hat{\mathbb{C}}_{t_r,-ij|t_s}(\mathbb{C}_{t_s,ij}), \mathbb{C}_{t_r,ij}\right)$ with the arbitrary $\mathbb{C}_{t_r,ij}$.

I will now define the value function of ij agreeing to a contract $\mathbb{C}_{t_s,ij}$ in Stage 2 of period t_s. The value function will depend in large part on the one-period-ahead value of beginning with a modified information state. The ex ante expected value to agent i of beginning the period t_s with information $\mathcal{I}_{t_s}$ with (potentially random) realized contracts $\hat{\mathbb{C}}_{t_s}(\mathcal{I}_{t_s})$, number of new contracts $\hat{R}_i(\mathcal{I}_{t_s})$, and otherwise equilibrium response functions of that form is:

$$V_i^{(1)}(\mathcal{I}_{t_s}) = E_{\mathcal{I}_{t_s}}\left[u_{it_s}\left(\hat{\mathbb{C}}_{t_s}(\mathcal{I}_{t_s})\right) + \sum_{j\neq i}\hat{p}_{ijt_s}(\mathcal{I}_{t_s})D_{t_s,ij}\left(\hat{\mathbb{C}}_{t_s}(\mathcal{I}_{t_s})\right) - r_i\hat{R}_i(\mathcal{I}_{t_s}) + \beta^{\frac{1}{m}}V_i^{(1)}(\mathcal{I}_{t_s+1})\right].$$

Also define $\hat{R}_{ijt_s}$ as an indicator for ij forming a new contract in t_s, and $\hat{B}_{ijt_s} \geq \hat{R}_{ijt_s}$ to be an indicator ij having the opportunity to bargain in a period t_s. Then the ex post expected value of $i < j$ choosing a contract $\mathbb{C}_{t_s,ij} \in \mathcal{C}_{ijt_s}/\mathbb{C}_{0,ijt_s}$ with implicit information $\mathcal{I}_{t_s}$ and associated price p_{ijt_s}, including the negotiation cost, is:

$$\begin{aligned} V_{i|ijt_s}^{(2,A)}\left(\mathbb{C}_{t_s,ij} \mid \hat{\mathbb{C}}_{t_s,-ij|t_s}\right) = {} & E_{t_s.2}\left[u_{it_s}\left(\left(\hat{\mathbb{C}}_{t_s,-ij|t_s}, \mathbb{C}_{t_s,ij}\right)\right) - r_i(\hat{R}_{it_s} - \hat{R}_{ijt_s} - r_i)\right] \\ & + E_{t_s.2}\left[\sum_{k\neq i} p_{ikt_s}D_{t_s,ij}\left(\left(\hat{\mathbb{C}}_{t_s,-ij|t_s}, \mathbb{C}_{t_s,ij}\right)\right)\right] \\ & + \beta^{\frac{1}{m}}E_{t_s.2}\left[V_i^{(1)}\left(\mathcal{I}_{t_s+1}\left(\left(\hat{\mathbb{C}}_{t_s,-ij|t_s}, \mathbb{C}_{t_s,ij}\right)\right)\right)\right]. \end{aligned}$$

The value to $j > i$ is similar, but with reindexing as needed. The value of ij disagreeing is:

$$V_{i|ijt_s}^{(2,D)}\left(\hat{\mathbb{C}}_{t_s,-ij|t_s}\right) = E_{t_s.2}\left[u_{it_s}\left(\left(\hat{\mathbb{C}}_{t_s,-ij|t_s}, \mathbb{C}_{0,ijt_s}\right)\right) - r_i(\hat{R}_{it_s} - \hat{R}_{ijt_s})\right]$$

$$+ E_{t_s.2}\left[\sum_{k\neq i,j} p_{ikt_s} D_{t_s,ij}\left(\left(\hat{\mathbb{C}}_{t_s,-ij|t_s}, \mathbb{C}_{0,ijt_s}\right)\right)\right]$$
$$+ \beta^{\frac{1}{m}} E_{t_s.2}\left[V_i^{(1)}\left(\mathcal{I}_{t_s+1}\left(\left(\hat{\mathbb{C}}_{t_s,-ij|t_s}, \mathbb{C}_{0,ijt_s}\right)\right)\right)\right].$$

Given these ex post value functions, I recursively define the ex ante value function for i in terms of the ij contract outcome. The three possibilities are renewing at equilibrium contract $\hat{\mathbb{C}}_{t_s,-ij|t_s}$ ($\hat{B}_{ijt_s} = 0$), successfully negotiating the (potentially random) equilibrium contract $\hat{\mathbb{C}}_{t_s,-ij|t_s}$ ($\hat{B}_{ijt_s} = \hat{R}_{ijt_s} = 1$), and disagreeing to the null contract $\hat{\mathbb{C}}_{0,ijt_s}$ ($\hat{B}_{ijt_s} = 1, \hat{R}_{ijt_s} = 0$). As a result, the ex ante expected NPV profit can be written as:

$$V_i^{(1)}\left(\mathcal{I}_{t_s}\right) = E_{\mathcal{I}_{t_s}}\left[(1-\hat{B}_{ijt_s}(1-\hat{R}_{ijt_s}))V_{i|ijt_s}^{(2,A)}\left(\hat{\mathbb{C}}_{t_s,ij} \mid \hat{\mathbb{C}}_{t_s,-ij|t_s}\right) + (1-\hat{B}_{ijt_s})r_i\right]$$
$$+ E_{\mathcal{I}_{t_s}}\left[\hat{B}_{ijt_s}(1-\hat{R}_{ijt_s})(1-\hat{R}_{ijt_s})V_{i|ijt_s}^{(2,D)}\left(\hat{\mathbb{C}}_{t_s,-ij|t_s}\right)\right].$$

A.2 Additional Proofs

Proof of Lemma 3. For this proof, let $\mathcal{G}$ be the set of games (S, v^D) satisfying Assumption 1, $v^D = 0$, and $S = \{s \in S : s \geq 0\}$. I write that the game $(\{0\}, 0) \in \mathcal{G}$, with $f(\{0\}, 0) = 0$ for any bargaining solution f.

I prove the weaker claim that if f satisfies WPO, IIA, and concavity for games in $\mathcal{G}$, then either f is proportional or f it utilitarian.

The proof generally follows Myerson (1981), so I proceed assuming the reader has a copy on hand. Let $\mathcal{G}$ be the set of games that satisfy Assumption 1 and do not allow ex post losses, and let f be a bargaining solution that satisfies WPO, IIA, and this proof's weaker form of concavity. Myerson's Theorem 1 shows that because $\mathcal{G}$ is a convex combination, if f is linear, then f is utilitarian.

Most of Myerson's results follow after some change in notation. His Lemma 1 through Lemma 6 follow after adjusting the notion of comprehensive convex hull to intersect with $\mathbb{R}^2_+$, and writing $M = \{f(G) : G \in \mathcal{G}\}$. Lemma 7 follows after adding a caveat that the claim only holds if $z = \lambda x(1-\lambda)y \geq 0$, and with some care verify the claim holds for $y = 0$.

Myerson's Lemma 8 requires more modification. Because f is not utilitarian, it continuous to be the case that there is a $v, u = f(v)$ such that $p \cdot v > p \cdot u$, where p is constructed in Myerson's Lemma 4. Then by Myerson's Lemma 7, take $\lambda = \frac{1}{\|u\|}$ and mix the game $H(v)$ with zero to obtain a game, with associated solution c satisfying $\|c\| = 1$. Without loss of generality I write that $u = c$. Then it clear by inspection that $u - c \in M$, because $f(\{0\}, 0) = 0 = u - c$.

Myerson's Lemmas 9 and 10 follow immediately. Lemma 11 follows with the caveat that the claim only holds if $u + \lambda d \geq 0$. Myerson's Lemma 12 completes the proof by showing that if $x \in M$, then $x = u + (p\dot{x} - p\dot{u})\, u$, which holds so long as $u + (p\dot{x} - p\dot{u})\, u \geq 0$. But by construction in this extension, $p\dot{u} = 1$, so that the only requirement is that $p\dot{x} \geq 0$. But $x \in M$, so there is an 0 such that $x = F(S, 0)$, and $0 \in M \cap S$, so that by Myerson's Lemma 5, $p\dot{x} \geq p\dot{0} = 0$. Therefore $u + (p \cdot x - p \cdot u)u \geq 0$, so that the result of Myerson's Lemma 12 holds and if f is not utilitarian, then f is proportional. □

Proof of Lemma 2. f_3 is not concave, so there exists an S, T, λ such that at least one player strictly prefers $\lambda f_3(S, v^D) + (1-\lambda) f_3(T, u^D)$ to $f_3(\lambda S + (1-\lambda)T, \lambda v^D + (1-\lambda)u^D)$. By strict inequality, it must be that $\lambda \in (0, 1)$.

Without loss of generality, assume player 1 strictly prefers the ex post game. By WPO of the ex ante solution, player 2 weakly prefers the ex ante game. Write $v_{EA} = f\left(\lambda S + (1-\lambda)T, \lambda v^D + (1-\lambda)u^D\right)$, $s^* = f_3(S, v^D)$, and $t^* = f_3(T, u^D)$, then $v_{EA,1} < \lambda s_1^* + (1-\lambda)t_1^*$ and $v_{EA,2} \geq \lambda s_2^* + (1-\lambda)t_2^*$.

Let s', t' be points in S, T such that $s' \geq v^D$, $t' \geq u^D$, $s_1' \geq s_1^*$, $s_2' \leq s_2^*$, $t_1' \geq s_1^*$, $t_2' \leq t_2^*$, and $\lambda s' + (1-\lambda)t' = v_{EA}$. Such a point must exist because $v_{EA} \in \lambda S + (1-\lambda)T$.

Now let S' be the comprehensive convex hull of s', s^*, and v^D, and let T' be the comprehensive convex hull of t', t^*, and u^D. The Pareto frontier of $\lambda S' + (1-\lambda)T'$ is the convex hull of $\lambda s' + (1-\lambda)t'$ and $\lambda s^* + (1-\lambda)t^*$. Let $R = 1$ correspond to playing S' and let $R = 0$ correspond to playing T'. Also let the utility functions u corresponding to mapping

$p \in [0,1]$ to points on the utility frontier, with $p = 1/3$ corresponding to s' and t' and $p = 2/3$ corresponding to s^* and p^*. (Note that it is possible for the frontier of utility to be a single point in one of these games.) Also let $\mathscr{P}$ be the singleton distribution of $Bernoulli(\lambda)$.

By constraint of Lemma 2, there exists an f' such that $f'(\lambda S' + (1-\lambda)T', \lambda v^D + (1-\lambda)u^D) = \lambda s' + (1-\lambda)t'$. Let b be a structure generated by $EA = 0$ bargaining under f, and let b' be a structure generated by $EA = 1$ bargaining under f'. Any equilibrium results in setting $p^* = 1/3$, so that $b \Leftrightarrow b'$. But f predicts setting $p^* = 2/3$ in the ex ante game, so that $f \not\Leftrightarrow f'$. Thus, $\mathcal{F}$ is not single-period identified. □

Proof of Proposition 4. By applying the step-by-step property T times and then canceling future (exogenously evolving) states, every (p, x) combination observed in a period t_0 must satisfy:

$$E\left[\sum_{t=0}^{T-1} \beta^t \prod_{0<h\leq t} (1+\phi_{t_0+h}) \mid x_{t_0}, \phi_{t_0}\right] p = E\left[\sum_{t=0}^{T-1} \beta^t \tilde{p}(x_{t_0+t}) \mid x_{t_0}, \phi_{t_0}\right].$$

Let x contain at least two prices, p_1^* and p_2^*. Then, by iterated expectations:

$$\frac{\sum_{t=0}^{T-1} \beta^t E\left[\prod_{0<h\leq t} (1+\phi_{t_0+h}) \mid x_{t_0}, \phi_{t_0}, p^* = p_1^*\right] p_1^*}{\sum_{t=0}^{T-1} \beta^t E\left[\prod_{0<h\leq t} (1+\phi_{t_0+h}) \mid x_{t_0}, \phi_{t_0}, p^* = p_2^*\right] p_2^*} = \frac{E\left[\sum_{t=0}^{T-1} \beta^t \tilde{p}(x_{t_0+t}) \mid x_{t_0}, \phi_{t_0}, p^* = p_1^*\right]}{E\left[\sum_{t=0}^{T-1} \beta^t \tilde{p}(x_{t_0+t}) \mid x_{t_0}, \phi_{t_0}, p^* = p_1^*\right]}.$$

This plus the constraints $E\left[\sum_{t=0}^{T-1} \beta^t \prod_{0<h\leq t} (1+\phi_{t_0+h}) p_{t_0}^* \mid x\right] = E\left[\sum_{t=0}^{T-1} \beta^t \tilde{p}_t(x) \mid x\right]$ provides $dim(x) + 1$ constraints for $dim(x) + 1$ parameters. Therefore, under appropriate rank conditions, all bargaining parameters are satisfied. □

Proof of Proposition 5. By Proposition 2, all bargaining parameters are identified if $T = 1$, so I proceed assuming $T > 1$.

Consider the distribution in new-contract periods t_0. Condition on a pair of x_{t_0}, ϕ_{t_0} from the sign constraint in the statement of Proposition 5. By applying the step-by-step property T times, canceling future (exogenously evolving) states, and applying iterated expectations, the must satisfy:

$$E\left[\sum_{t=t_0}^{t_0+T-1}\beta^{t-t_0}\left(\prod_{t_0<v\le t}\phi_v\right)p^*_{t_0}\mid x_{t_0},\phi_{t_0},z_{t_0}\right]=E\left[\sum_{t=t_0}^{t_0+T-1}\beta^{t-t_0}\tilde{p}(x_t)\mid x_{t_0},\phi_{t_0},z_{t_0}\right]$$
$$=E\left[\sum_{t=t_0}^{t_0+T-1}\beta^{t-t_0}\tilde{p}(x_t)\mid x_{t_0},\phi_{t_0}\mid x_{t_0},\phi_{t_0}\right]=\text{“}C\text{.”}$$

Thus:

$$C=E\left[\left(\sum_{t=t_0}^{t_0+T-1}\beta^{t-t_0}\left(\prod_{t_0<v\le t}\phi_v\right)\right)p^*_{t_0}\mid x_{t_0},\phi_{t_0},z_{t_0}\right].$$

Subtraction between two levels of z_{t_0} yields identification of β. □

Proof of Theorem 2. Consider a period t_0's Stage 2 subgame in which $i<j$ form a new contract with positive probability. For simplicity, I proceed assuming there is a pure strategies equilibrium, and that in every subgame that can be reached after t_0, the expected number of subsequent ij agreements is infinite.[3] Also for simplicity, assume that a realized contract $\mathbb{C}_{t_s,ij}$ will be unchanged until the next period in which ij have the opportunity to negotiate. I write the anticipated non-ij contracts in period t_s as $\mathbb{C}_{t_s,-ij}$.

I consider two off-equilibrium paths: an agreement-followed-by-impasse path (superscripted A), and an immediate-impasse path (superscripted D). I write $t_0^A=t_0^D=t_0$, and I write t_a^A and t_a^D for the (potentially random) a^{th} future period in which ij defect from equilibrium and reach the null contract on that path.

For $d\ge 0$, define $V^{(2,E)}_{k,(d)^D}$ to be the (potentially random) expected NPV to player k of ij existing impasse in period t_d^D at the other-pair subgame equilibrium contracts $\hat{\mathbb{C}}_{t_d^D,-ij}$. Let the subgame equilibrium have ij choose $\hat{\mathbb{C}}_{t_d^D,ij}$. Also define $V^{(2,I)}_{k,(d)^D}$ to be the expected NPV of remaining in impasse, and define $GFT^{(2,E)}_{(d)^D}\equiv V^{(2,E)}_{i,(d)^D}+V^{(2,E)}_{j,(d)^D}-V^{(2,I)}_{i,(d)^D}-V^{(2,I)}_{j,d}$ to be the joint subgame gains from trade. Because ij follow the Kalai proportional solution, $\hat{C}_{t_d^D,ij}$ is

[3] In estimation, it is infeasible to include an infinite number of bargains without imposing strong assumptions like steady state behavior. I therefore impose a finite horizon model in estimation as an approximation to an infinite-horizon model.

chosen to maximize joint gains from trade subject to:

$$(\tau_{ij})\left(V^{(2,E)}_{i,(d)^D}\left(\left(\hat{\mathbb{C}}_{t^D_d,-ij},\hat{\mathbb{C}}_{t^D_d,ij}\right)\right)-V^{(2,I)}_{i,(d)^D}\right)=(1-\tau_{ij})\left(V^{(2,D)}_{j,d}\left(\left(\hat{\mathbb{C}}_{t^D_d,-ij},\hat{\mathbb{C}}_{t^D_d,ij}\right)\right)-V^{(2,I)}_{j,d}\right).$$

Then:

$$\begin{aligned}
V^{(2,I)}_{i,(d)^D} &= E_{t^D_d}\left[\sum_{t^D_d\le t_s<t^D_{d+1}}\beta^{\frac{t_s-t^D_d}{m}}\left(\hat{\pi}^D_{it_s}+\hat{T}^D_{it_s}\right)+\beta^{\frac{t^D_{d+1}-t^D_d}{m}}V^{(2,E)}_{i,(d+1)^D}\left(\left(\hat{\mathbb{C}}_{t^D_{d+1},-ij},\hat{\mathbb{C}}_{t^D_{d+1},ij}\right)\right)\right]\\
&= E_{t^D_d}\left[\sum_{t^D_d\le t_s<t^D_{d+1}}\beta^{\frac{t_s-t^D_d}{m}}\left(\hat{\pi}^D_{it_s}+\hat{T}^D_{it_s}\right)+\beta^{\frac{t^D_{d+1}-t^D_d}{m}}\left(V^{(2,I)}_{i,(d+1)^D}+(1-\tau_{ij})GFT^{(2,E)}_{(d+1)^D}\right)\right]\\
&= E_{t^D_d}\left[\sum_{t_s\ge t^D_d}\beta^{\frac{t_s-t^D_d}{m}}\left(\hat{\pi}^D_{it_s}+\hat{T}^D_{it_s}\right)+\sum_{h>0}(1-\tau_{ij})\beta^{\frac{t^D_{d+h}-t^D_d}{m}}GFT^{(2,E)}_{(d+h)^D}+\lim_{h\to\infty}\beta^{\frac{t^D_{d+h}-t^D_d}{m}}V^{(2,I)}_{i,(d+h)^D}\right]\\
&= E_{t^D_d}\left[\sum_{t_s\ge t^D_d}\beta^{\frac{t_s-t^D_d}{m}}\left(\hat{\pi}^D_{it_s}+\hat{T}^D_{it_s}\right)+\sum_{h>0}(1-\tau_{ij})\beta^{\frac{t^D_{d+h}-t^D_d}{m}}GFT^{(2,E)}_{(d+h)^D}\right],
\end{aligned}\tag{8}$$

where the final line uses $\lim_{h\to\infty}\beta^{\frac{t^D_{d+h}-t^D_d}{m}}V^{(2,I)}_{i,(d+h)^D}=0$ under transversality conditions.

I iteratively expand using Equation (8) and obtain:

$$\begin{aligned}
V^{(2,I)}_{i,(d)^D} &= E_{t^D_d}\left[\sum_{t_s\ge t^D_d}\beta^{\frac{t_s-t^D_d}{m}}\left(\hat{\pi}^D_{it_s}+\hat{T}^D_{it_s}\right)+\sum_{h>0}(1-\tau_{ij})\beta^{\frac{t^D_{d+h}-t^D_d}{m}}GFT^{(2,E)}_{(d+h)^D}\right]\\
V^{(2,I)}_{j,(d)^D} &= E_{t^D_d}\left[\sum_{t_s\ge t^D_d}\beta^{\frac{t_s-t^D_d}{m}}\left(\hat{\pi}^D_{jt_s}+\hat{T}^D_{jt_s}\right)+\sum_{h>0}(\tau_{ij})\beta^{\frac{t^D_{d+h}-t^D_d}{m}}GFT^{(2,E)}_{(d+h)^D}\right].
\end{aligned}$$

Analogous results hold to characterize $V^{(2,I)}_{i,(d)^A}=V^{(2,E)}_{i,(d)^D}\left(\left(\hat{\mathbb{C}}_{t^D_d,-ij},\hat{\mathbb{C}}_{t^D_d,ij}\right)\right)$ and $V^{(2,I)}_{j,(d)^A}=V^{(2,E)}_{j,(d)^D}\left(\left(\hat{\mathbb{C}}_{t^D_d,-ij},\hat{\mathbb{C}}_{t^D_d,ij}\right)\right)$, except that in periods t_0 through (t^A_1-1), ij have a contract on the A path, so that ij profits are $\hat{\pi}^A+\hat{T}^A+[\Delta_{ij}\pi]+[\Delta_{ij}T_{-.}]+pD_{ijt}$ instead of $\hat{\pi}^A+\hat{T}^A$ and ij pay an additional negotiation cost in period t_0.

Substituting these results together, characterizing the surplus split of the $t_0=t^D_0=t^A_0$

bargain, and moving around terms, I obtain that for any equilibrium agreement:

$$
\begin{aligned}
E_{t_0}\left[\sum_{t_0 \le t_s < t_q^A} \beta^{\frac{t_s-t_0}{m}} p^*_{ijt_s} D_{ijt_s}\right] &= E_{t_0}\left[\sum_{t_0 \le t_s < t_q^A} \beta^{\frac{t_s-t_0}{m}} \left\{ \begin{array}{cc} -\tau_{ij} & ([\Delta_{ij} u_{it_s}] + [\Delta_{ij} T_{it_s,-j}]) \\ +(1-\tau_{ij}) & ([\Delta_{ij} u_{jt_s}] + [\Delta_{ij} T_{jt_s,-i}]) \end{array} \right\}\right] \\
&\quad + (\tau_{ij} r_i - (1-\tau_{ij}) r_j) \\
&\quad + E_{t_0}\left[\sum_{t_0 \le t_s} \beta^{\frac{t_s-t_0}{m}} \left\{ \begin{array}{cc} -(\tau_{ij}) & \left(\hat{\pi}^A_{it_s} + \hat{T}^A_{it_s} - \hat{\pi}^D_{it_s} + \hat{T}^D_{it_s}\right) \\ +(1-\tau_{ij}) & \left(\hat{\pi}^A_{jt_s} + \hat{T}^A_{jt_s} - \hat{\pi}^D_{jt_s} + \hat{T}^D_{jt_s}]\right) \end{array} \right\}\right] \\
&= \text{Pay}_{NiN} + \text{Pay}_{NC} + \text{Pay}_{IRT},
\end{aligned}
$$

which is the desired equality. □

A.3 Tables and Figures

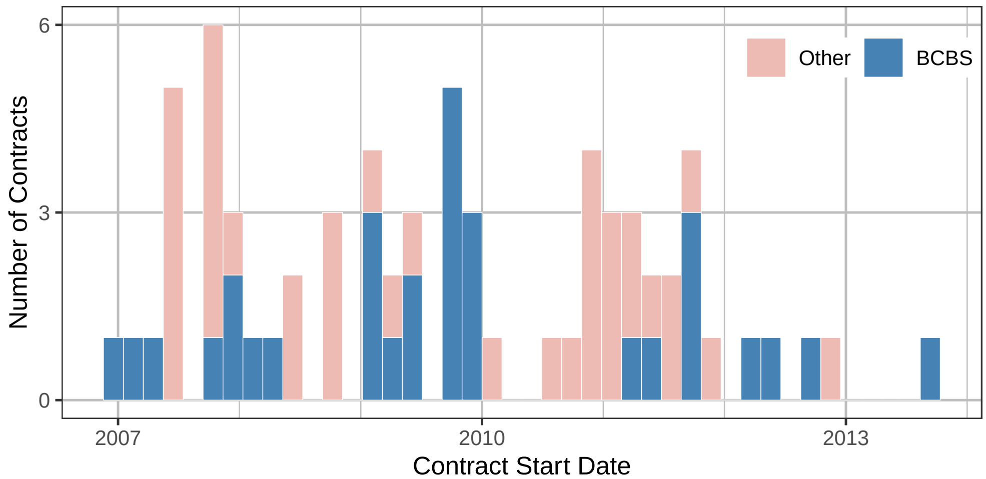

Figure 4: Contracts are formed at different times. Histogram of contract start dates for contracts used in the estimation sample and introduced in 2007–2014 for Highmark BCBS (blue) and other modeled insurers (pink). Vertical lines indicate January 1 of a given year. Contracts were not systematically introduced on the same dates.

Table 2: Additional estimated bargaining parameters. BCBS parameters correspond to Highmark BCBS. "Data" corresponds to average difference between MLR-implied costs per life and estimated average inpatient payments per life insured, and would exactly set the MLR moment to zero for the myopic and forward-looking models. The r^M net negotiation costs are close to their starting point of $10,000 and may weakly identified or unidentified.

	Parameter (τ^{Size} Estimated)							
	η_{BCBS}	η_{HPUOV}	η_{Aetna}	$\eta_{UnitedHealth}$	η_{Cigna}	$\eta_{Carelink}$	r^M_{yBCBS}	r^M_{nBCBS}
Only-2015	3657***	3404***	3658***	2008***	4627***	3139***	10000***	9999***
(Nash/Kalai)	(45)	(85)	(116)	(29)	(32)	(39)	(2614)	(1441)
Myopic	4640***	4036***	3659***	3197***	4624***	3139***	10000***	10000***
(Nash/Kalai)	(14)	(650)	(37)	(374)	(26)	(463)	(1444)	(1)
Forward-Looking	4638***	3631***	3660***	3284***	4626***	3140***	9999***	9999***
($Pay_{IRT} = 0$)	(130)	(302)	(37)	(69)	(30)	(45)	(29)	(65)
Data	3600	3356	3554	1999	4635	3114		

Note: *p<0.1; **p<0.05; ***p<0.01