A Microfoundation for the Nash-in-Kalai Model

Jacob Dorn*

June 12, 2025

Abstract

This note presents a microfoundation for the Nash-in-Kalai model proposed by the companion paper Dorn (2025). The microfoundation is a demands game with revocation costs based on Dutta (2012). As opportunities to contract become instantaneous and the marginal cost of the first dollar of concession tend to infinity, gains from trade tend to zero and the bargaining solution tends to an instantaneous (and by extension a discrete) dynamic Kalai proportional bargaining solution. The game extends Dutta's setup to include repeated bargaining.

This note presents a microfoundation for the Nash-in-Kalai model (Dorn, 2025): a Nash equilibrium in recursively defined Kalai proportional bargains. Collard-Wexler et al. (2019) present a microfoundation for Nash-in-Nash, but it only applies to the transferable utility case in which Nash-in-Nash and Nash-in-Kalai coincide.

Unlike Nash bargaining, the predicted outcome of Kalai proportional bargaining depends on the scale of gains from trade. As a result, the microfoundation cannot depend solely on von Neumann-Morgenstern utility, which is scale-invariant. Instead, this microfoundation builds on Dutta (2012) to leverage concession costs to fix relative scales of utility. Kalai proportional bargaining can also be microfounded through an arbitration game (Bossert, 1994), while Nash bargaining can be microfounded through demands (Nash, 1953), alternating offers (Binmore et al., 1986), and other games; both can be microfounded through liquidity constraints (Hu and Rocheteau, 2020).

*This material is based upon work supported by the National Science Foundation Graduate Research Fellowship Program under Grant No. DGE-2039656 and by grant T32 HS026116 from the Agency for Healthcare Research and Quality. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation or the Agency for Healthcare Research and Quality. All errors are my own.

The game is a series of periods in which contracts can be reached at one of two stages: Nash bargaining between jointly feasible price demands, or the opportunity to make a costly concession after revelation of jointly infeasible demands. Agreements are made between the two sides of the market: hospitals (i.e. suppliers), who always prefer higher prices, and insurers (i.e. retailers), who always prefer lower prices.

I show two main results for this game. First, I show that as the length of a period tends to zero, the predicted agreement tends to the Kalai proportional solution, with the ratio of bargaining weights equal to the ratio of first derivatives in the limit. This result is essentially an extension of Dutta (2012) to multilateral games. Second, I show that the implied agreement is the same if the value of disagreement is replaced by Binmore et al. (1989)'s impasse point in discrete time. This result essentially follows by the Kalai proportional bargaining solution's step-by-step property (Kalai, 1977, Roth, 1979, Dorn, 2025), and shows that the continuous-time model implies a Nash-in-Kalai representation in discrete time.

I first describe a sequence of demands games indexed by n. I then show that as the frequency of bargaining opportunities tends to infinity, any pure strategies Markov Perfect equilibrium of the demands game tends to Kalai proportional bargaining.[1] I then show that a corresponding sequence of discrete time bargaining solutions of the instantaneous bargaining game correspond to the dynamic Kalai proportional bargaining solution in discrete time.

1 The Demands Game

The sequence of discretely timed games is indexed by n. I avoid notation for period lengths by instead showing that as gains from trade tend to zero in n, the implied solution approaches Nash-in-Kalai uniformly. Taking period lengths to zero generally implies increasingly small

[1] I mainly use the Markov assumption to prevent renegotiation in equilibrium. Without it, the sides could sustain an equilibrium with painful concessions by punishing forming the same contracts without concession. Dutta (2024) shows that the microfoundation only requires renegotiation-proof strategies when repeatedly attempting to bargain over a fixed asset. It is likely that this property relates to Myerson (1981)'s notion of concavity in planning: under Kalai proportional or utilitarian bargaining, ex ante negotiations produce a weakly better expected value than ex post negotiations, so that it can never be strictly Pareto-improving to replace the outcome of ex ante bargaining with the outcome of ex post bargaining.

games from trade, and therefore implies a Nash-in-Kalai limit.

In game n, time is discrete and runs from $t = 0$ to infinity. In period t, a contract structure $\mathbb{C}_t$ emerges. In period t, the contract structure which emerges is a set of lengths hospital i-insurer j lengths $\ell_{ijt,(n)}$ and a set of $i-j$ prices $p_{ijt,(n)} \subseteq \mathcal{P}$, where $\mathcal{P}$ is a closed, convex subset of $\mathbb{R}$.[2] If i and j do not reach a contract, then $\ell_{ijt,(n)} = p_{ijt,(n)} = 0$. I write the set of contracts that emerge as $\mathbb{C}_t$. I assume that at every stage of negotiations, upstream hospitals prefer higher prices while downstream insurers prefer lower prices. A contract corresponds to a constant per-unit rate in place for the full length of the contract: for example, lump-sum payments would fit in the model as a price per period amortized over the contract.

There is no uncertainty. The set of insurer and hospital indices remains the same in every game. I assume that if i and j contract in period t, the agreement will be in place for the known, exogenous value $\ell^*_{ijt,(n)}$.

Timing in period t is as follows:

1. The board of directors of every hospital i and insurer j meet with their delegates, who will simultaneously bargain with every potential partner with whom they do not have an agreement.

    - Hospital delegates and insurer delegates choose a price demand to state publicly. The demand is chosen to maximize a weighted average of their employer's net present value profits and a personal concession cost they will face if they agree to a contract that is worse than their demand. The hospital delegates demand a minimum price $\underline{p}^H_{Demand}$ and the insurer delegates demand a maximum price $\bar{p}^M_{Demand}$.

2. The corresponding delegates for each ij pair without a contract in place from the previous period simultaneously meet with their authorized demands. If an ijt pair has jointly feasible demands $\underline{p}^H_{ijt,Demand} \leq \bar{p}^M_{ijt,Demand}$, the delegates reach a jointly feasible

[2]The game immediately generalizes to other vertical markets by treating hospitals as an upstream market, insurers as a downstream market, and prices as a real-valued numeraire the sides bargain over.

contract by Nash bargaining over firm profits, treating their demands as disagreement points and taking equilibrium strategies of other pairs as given. (If this is not well-defined, the pair choose the average demand.)

3. Each delegate of an ij pair without a contract has the opportunity to concede to the other side's demands. Conceding means adopting the other delegate's demand. Without loss of generality, I write concession costs in units of employer net present value profits. Conceding has an associated cost of $c^H_{ijt,(n)}(\underline{p}^H_{Demand} - \bar{p}^M_{Demand})$ and $c^H_{ijt,(n)}(\underline{p}^H_{Demand} - \bar{p}^M_{Demand})$. The concession cost functions are continuous, strictly increasing for positive concessions, equal to zero and have an infinite right-differentiable at zero, and are uniformly lower-bounded by functions with these properties.

4. Each ij pair without a contract meets simultaneously.

   - If the new demands are jointly feasible, they reenter the same joint bargaining process as in Stage 2.
   - If the new demands are jointly infeasible, ij do not form a contract in period t.

5. The hospitals and insurers obtain flow profits: $v^H_{it}(\mathbb{C}_t, R_t) = \pi^H_{it,(n)}(\mathbb{C}_t) + r^H_{i,(n)} R_{ijt}$ for hospital i where r^H_i is any new contract negotiation cost, and analogously $v^M_{jt}(\mathbb{C}_t, R_t) = \pi^M_{jt,(n)}(\mathbb{C}_t) + r^M_{j,(n)} R_{ijt}$ for insurer j.

The outcome of each stage is always immediately announced to all parties.

When the delegates arrive with jointly-unachievable demands, they play a one-shot game with a simultaneous payoff matrix adapted from Dutta and depicted in Appendix Table 1. In Appendix Table 1, I write the value functions with agreement (at the anticipated concession decisions) as V^H and V^M and the value with disagreement as V^H_D and V^M_D. The agreement value functions include any effect of the negotiated contract on any other agreements reached through concession in the period. I later show that there is no concession in equilibrium, so that the relevant value functions are also the value functions at the equilibrium simultaneous agreements. If concession costs are too high, neither delegate will be willing to concede to a contract that only improves their employer's profits slightly. The infinite right-derivative at zero ensures that if the demands are close enough, then both delegates will prefer to concede despite the incurred concession cost.

Table 1: After incompatible demands in Stage 2 ($\underline{p}^H > \bar{p}^M$), payoffs in Stage 3 depending on whether hospital delegate (rows) and insurer delegate (columns) concedes or sticks to their initial demands. Table is adapted from Dutta (2024). I omit the ijt subscripts for brevity. p^* is the hypothetical Nash bargained price if both delegates concede and the demands are reversed.

	Concede (C)	Stick (S)
C	$(V^H(p^*) - c^H(\underline{p}^H - \bar{p}^M), V^M(p^*) - c^M(\underline{p}^H - \bar{p}^M))$	$(V^H(\underline{p}^H) - c^H(\underline{p}^H - \bar{p}^M), V^M(\underline{p}^H))$
S	$(V^H(\bar{p}^M), V^M(\bar{p}^M) - c^M(\underline{p}^H - \bar{p}^M))$	(V_D^H, V_D^M)

The concession costs constrain the contracts that can emerge in equilibrium. Suppose jointly compatible demands will lead to the hospital getting the most favorable deal: a take it or leave it outcome that gives the hospital all surplus and leaves the insurer with their disagreement value. Consider a subgame in which the insurer demands a slightly better deal. The insurer's delegate will not pay a concession cost to concede and obtain the firm's disagreement value. On the other hand, if the new demand is close enough to the disagreement value, the hospital's delegate will prefer to concede (an arbitrarily small cost) to avoid disagreement (a fixed cost to the hospital given n). The same logic can be applied to lopsided deals: the concession costs constrain how much surplus each side can obtain. The higher one side's concession costs, the better of a deal they guarantee themselves in equilibrium. In the limit as the game becomes instantaneous and gains from trade tend to zero, the constraint becomes driven by the derivative of the cost functions at zero.

The particular form of concession costs enables a scale varying solution in the limit. If the ratio of marginal concession costs changed over time, the solution would be instantaneous but not discrete Kalai proportional bargaining. If the concession costs were a function of value conceded, it appears the equilibrium would correspond to Nash bargaining at the margin in a similar manner to Coles and Muthoo (2003). If the concession costs were borne by the delegates but set by firms that were indifferent to conceding, then there would be equilibria with concession and the concession costs might not have any bite.

The model could be generalized in a few directions at the cost of additional notation. I use Nash bargaining in Stage 2 to be tongue-in-cheek. Any other bargaining solution would work. I make contract lengths exogenous to ensure that the space of feasible contract values is convex. The model could likely be extended to enable endogenous contract lengths. Under this game and Dutta (2024)'s game, a delegate pays the same concession cost whether or not the other side concedes; in Dutta (2012)'s original game, the concession cost is paid based on the difference between demanded and realized price, with the same result.

I now write out value functions for the realized contract state. I will assume players follow Markov strategies, so that value functions only depend on realized contracts and negotiation costs (i.e. realized contracts and the previous period's realized contracts). Suppose the players follow strategies $\hat{\sigma}$ which generate period $t+1$ contracts $\hat{\sigma}_{t+1}(\mathbb{C}_t)$. I define the corresponding value functions as:

$$
\begin{aligned}
V^H_{it,(n)}(\mathbb{C}_t \mid \mathbb{C}_{t-1}) &= \frac{\pi^H_{it,(n)}(\mathbb{C}_t) - \sum_j r^H_{i,(n)} R_{ijt} + \beta_{(n)} V^H_{it+1,(n)}(\hat{\sigma}_{t+1}(\mathbb{C}_t) \mid \mathbb{C}_t)}{1-\beta} \\
V^M_{it,(n)}(\mathbb{C}_t \mid \mathbb{C}_{t-1}) &= \frac{\pi^M_{jt,(n)}(\mathbb{C}_t) - \sum_i r^M_{j,(n)} R_{ijt} + \beta_{(n)} V^M_{jt+1,(n)}(\hat{\sigma}_{t+1}(\mathbb{C}_t) \mid \mathbb{C}_t)}{1-\beta}.
\end{aligned}
$$

In that equation, R_{ijt} is an indicator for ij forming a new contract in period t.

2 Continuous-Time Results

I will assume some structure on the value functions which I expect to hold in many vertical market bargaining models. I will make use of the value of ij deviating to a new contract p while holding fixed the outcome of other bargains. I write these unilateral deviation value functions as $V^H_{ijt}(p_{ijt} \mid \hat{\sigma}, \mathbb{C}_{t-1})$ and $V^M_{ijt}(p \mid \hat{\sigma}, \mathbb{C}_{t-1})$. It is not obvious at this stage that unilateral-deviation value functions are the right deviation value in this bargaining game. I show there is no concession in equilibrium, so that these are the relevant value functions.

Assumption 1 (Monotone and differentiable value function)**.** All strategies are Markov. When bargaining in Stage 2 or choosing whether or not to concede in Item 3, hospitals strictly prefer higher prices and insurers strictly prefer lower prices inclusive of any response through subsequent concession decisions and negotiations in period t. The expected value functions of a bargained initial price p at the expected other equilibrium contracts in the same period is written as $V(p \mid \hat{\sigma}, \mathbb{C}_{t-1})$ and is differentiable with bounded derivatives as follows:

$$0 < \varepsilon B \leq \frac{-\partial V_{ijt}^{H}(p_{ijt} \mid \hat{\sigma}, \mathbb{C}_{t-1})}{\partial p_{ijt}}, \frac{\partial V_{ijt}^{H}(p_{ijt} \mid \hat{\sigma}, \mathbb{C}_{t-1})}{\partial p_{ijt}} \leq B$$

for uniformly bounding constants $\varepsilon, B > 0$.

Since one side strictly prefers higher prices and the other side strictly prefers lower prices, Assumption 1 allows me to write the value concession game as a value-based price concession game. Assumption 1 could be relaxed to a Lipschitz continuity-type assumption.

The substantive idea of this assumption is that prices have monotonic effects. The first half that includes downstream effects rules out the delegates choosing to form a contract through jointly feasible demands in order to sustain an equilibrium in other simultaneous demands. Under Assumption 1, if a hospital and insurer arrive with jointly feasible demands, they could do better by deviating to the other's demand despite any effects on the downstream contracts.

The value function derivative component of Assumption 1 ensures the hold-fixed contract deviation value functions are differentiable. As a result, the value functions are invertible in bargained prices. For example, if the price domain $\mathcal{P}$ includes only weakly positive prices, then equilibrium hospitals generally prefer strictly higher prices and insurers prefer strictly lower prices. The property is likely to hold in other settings if which higher prices have positive spillovers on other prices for appropriately defined price domains $\mathcal{P}$.

Lemma 1. *Under Assumption 1, for every game n, subgame $\mathbb{C}_{t-1}$, triplet ijt without a contract in place under that subgame, and associated equilibrium strategies $\hat{\sigma}_{(n)}$, there are prices $p^H_{ijt,(n),D}$ and $p^M_{ijt,(n),D}$ that make the hospital and insurer, respectively, indifferent between agreement at that price and disagreement under the expected contracts formed by other pairs in equilibrium.*

I add an assumption to rule out certain nuisance behavior.

Assumption 2. If hjt do not reach a contract in a period t subgame and ijt do reach a contract through negotiation after initial jointly-feasible demands, then hnt continue to not reach a contract if either i or j strengthens their demand.

Assumption 2 rules out a certain edge case in which pairs reach a contract through Nash bargaining between jointly feasible contracts, but neither side can make a stronger demand because it would lead to an anticipated contract that changes other pairs' concession decision. Without Assumption 2, there is no concession in equilibrium, but the contract outcome may be driven by the effect on others' concession decisions in the same period. The content is minimal if, as in my setting, equilibrium networks are fairly complete.

The following lemma shows that there is no concession in equilibrium. As a result, in any equilibrium the firms must negotiate over an individual contract in a way that is optimal taking the outcome of other bargains as fixed. The best deviation over all demands is at least as good as the best deviation over a single demand.

Lemma 2. *Under Assumptions 1 and 2, for every game n with a pure strategy Markov perfect equilibrium $\hat{\sigma}_{(n)}$, every subgame contract is formed through equal demands.*

I obtain Kalai bargaining strategies as the game tends to instantaneous offers and the ratio of first marginal costs tends to some proportion.

Assumption 3 (Sequence of PSMPE tending to instantaneous)**.** As the game index n tends to infinity, bargaining becomes instantaneous in the sense that $\beta(n) \to 1$ and the effect of disagreement becomes negligible: $\max\{V^H(p_D^M)-V^H(p_D^H),0\},\max\{V^M(p_D^H)-V^M(p_D^M),0\} = o_n(1)$ uniformly in subgames and potential bargainers.

This is plausible in many games: the difference between agreeing to a contract now and waiting a second and agreeing to essentially the same contract should be essentially nil.

Assumption 4 (First marginal costs tend to proportional)**.** As the game index n tends to infinity, the ratio of first marginal costs tend to a fixed proportion in the sense that there are finite $w_i^H, w_j^M > 0$ and a sequence of $\delta_n \to 0$ such that $\max_{ij} \sup_{x\in(0,\max\{p^M_{ijt,D}-p^H_{ijt,D},\delta_n\}]} \frac{c_{ijt,(n)^M}(x)}{c_{ijt,(n)^H}(x)} - \frac{w_i^H}{w_j^M} = o_n(1)$.

The cost proportionality around zero is important to extending the Kalai proportional results from instantaneous to discrete time. It will be important for the microfoundation that as the contract shifts under impasse, the ratio of first marginal costs between the hospital and insurer retain the same proportions. It is not important that the costs be proportional on a price scale in particular, so long as the costs are in the same fixed units — costs could be formed under a payment scale and I would obtain the same result.

The following result follows from an adapted version of Dutta (2012)'s argument.

Proposition 1. *Suppose Assumptions 1, 2, 3, and 4 hold. Then the bargains tend to an instantaneous Kalai proportional solution:*

$$\sup_{\mathbb{C}_{t-1},R_{ijt,(n)}=1,p_D^M>p_D^H} \left| \frac{V_{ijt}^M\left(\hat{p}_{ijt}(\mathbb{C}_{t-1}) \mid \hat{\mathbb{C}}_{t-ij,(n)}\right) - V_{ijt}^M\left(p^M_{ijt,D} \mid \hat{\mathbb{C}}_{t-ij,(n)}\right)}{V_{ijt}^H\left(\hat{p}_{ijt}(\mathbb{C}_{t-1}) \mid \hat{\mathbb{C}}_{t-ij,(n)}\right) - V_{ijt}^H\left(p^H_{ijt,D} \mid \hat{\mathbb{C}}_{t-ij,(n)}\right)} - \frac{w_i^H}{w_j^M} \right| \to^n 0.$$

I offer the following intuition. The concession costs ensure that in every pure strategy Nash equilibrium, there is no concession and both sides get sufficiently more than their disagreement value that they cannot guarantee a better outcome by demanding more. As the game tends to instantaneous, the gains from trade relative to waiting a period become small and the infinite first marginal costs become binding. The particular form of the constraint is that the ratio of gains from trade tend to the ratio of first marginal costs. This is a scale varying solution concept because the concession costs are made based on prices rather than profits; fixing the ratio of concession costs fixes the relative value of profits.

3 Discrete-Time Results

The result in Proposition 1 gives a result about disagreeing over an ignorable period of time. Kalai proportional bargaining has a special path independence property that makes this instantaneous-bargaining limit extend to discrete time.

Proposition 2. *Suppose Assumptions 1, 2, 3, and 4 hold. Let $\tilde{V}^H_{ijt,(n)}(0 \mid \mathbb{C}_{t-1})$ and $\tilde{V}^M_{ijt,(n)}(0 \mid \mathbb{C}_{t-1})$ be the expected values if ij disagree in period t and remain in impasse until the next period where another pair forms a contract. Suppose the value of agreement relative to impasse is bounded. Then the bargains tend to an* discrete-*time Kalai proportional solution:*

$$\sup_{\mathbb{C}_{t-1}, R_{ijt,(n)}=1, V^M(p)>V^M(0)} \left| \frac{V^H_{ijt,(n)}(\hat{\sigma}(\mathbb{C}_{t-1})) - \tilde{V}^H_{ijt,(n)}(0 \mid \mathbb{C}_{t-1})}{V^M_{ijt,(n)}(\hat{\sigma}(\mathbb{C}_{t-1})) - \tilde{V}^M_{ijt,(n)}(0 \mid \mathbb{C}_{t-1})} - \tau_{ij} \right| \to^n 0.$$

Proposition 2 justifies using a discrete-timed dynamic Kalai proportional bargaining model even when real bargaining is conducted in continuous time: both the value of agreement and of impasse are defined in discrete time. This justifies theoretical and empirical analysis in discrete time.

Only Kalai proportional bargaining justifies estimating a discrete-time bargaining model with a continuous timed underlying microfoundation in general nonstationary games. A bargaining solution that generally returns the same contract after adding an additional post-disagreement chance to bargain must have proportional character (Roth, 1979). As a result, a comparable result for Nash-in-Nash bargaining would generally yield a differential equation at the margin (Coles and Muthoo, 2003, O'Neill et al., 2004). Nash-in-Nash bargaining in nonstationary environments with access to lump-sum transfers might be able to be microfounded, but only because Nash bargaining with access to lump-sum transfers is transferable utility, and therefore has the same predictions as Kalai proportional bargaining.

References

Binmore, K., Rubinstein, A., and Wolinsky, A. (1986). The Nash bargaining solution in economic modelling. *The RAND Journal of Economics*, 17(2):176–188.

Binmore, K., Shaked, A., and Sutton, J. (1989). An outside option experiment. *The Quarterly Journal of Economics*, 104(4):753–770.

Bossert, W. (1994). Disagreement point monotonicity, transfer responsiveness, and the egalitarian bargaining solution. *Social Choice and Welfare*, 11(4):381–392.

Coles, M. G. and Muthoo, A. (2003). Bargaining in a non-stationary environment. *Journal of Economic Theory*, 109(1):70–89.

Collard-Wexler, A., Gowrisankaran, G., and Lee, R. S. (2019). "Nash-in-Nash" bargaining: A microfoundation for applied work. *Journal of Political Economy*, 127(1):163–195.

Dorn, J. (2025). The Nash-in-Kalai model for estimation with dynamic bargaining and nontransferable utility.

Dutta, R. (2012). Bargaining with revoking costs. *Games and Economic Behavior*, 74(1):144–153.

Dutta, R. (2024). Bargaining as a struggle between competing attempts at commitment. *Review of Economic Studies*, 91(5):2771–2805.

Hu, T.-W. and Rocheteau, G. (2020). Bargaining under liquidity constraints: Unified strategic foundations of the Nash and Kalai solutions. *Journal of Economic Theory*, 189:105098.

Kalai, E. (1977). Proportional solutions to bargaining situations: Interpersonal utility comparisons. *Econometrica*, 45(7):1623–1630.

Myerson, R. B. (1981). Utilitarianism, egalitarianism, and the timing effect in social choice problems. *Econometrica*, 49(4):883–897.

Nash, J. (1953). Two-person cooperative games. *Econometrica*, 21(1):128–140.

O'Neill, B., Samet, D., Wiener, Z., and Winter, E. (2004). Bargaining with an agenda. *Games and Economic Behavior*, 48(1):139–153.

Roth, A. E. (1979). Proportional solutions to the bargaining problem. *Econometrica*, 47(3):775–778.

A Lemmas and Proofs

A.1 Lemmas

Lemma 3 (Dutta (2012), Proposition 2)**.** *Suppose $\hat{\sigma}$ is a pure strategy Markov perfect equilibrium of the game I have described in Proposition 1 under Assumption 1 but not necessarily 3 and 4. Suppose i and j could form a strictly Pareto-improving contract in period t. For brevity, I omit the $ijt,(n)$ subscripts. Then there is a unique $y_1, y_2 \in (0,1)$ that satisfies $y_1 + y_2 \geq 1$ and the following property about gains relative to disagreement*

$$V^H\left(y_2 p_D^H + (1-y_2)p_D^M\right) - V^H(p_D^H) = c^H\left((p_D^M - p_D^H)(y_1+y_2-1)\right)$$
$$V^M\left(y_1 p_D^M + (1-y_1)p_D^H\right) - V^M(p_D^M) = c^M\left((p_D^M - p_D^H)(y_1+y_2-1)\right),$$

then the pair (y_1, y_2) is unique.

Intuitively, y_1 and y_2 both decrease the left-hand sides to zero but increase the right-hand sides, so there should be a fixed point. In addition, only one of y_1 and y_2 appear on the left-hand side of any given equation. Consider the function $\hat{y}_1(y_2)$ that chooses a $\hat{y}_1$ to hold the first equation with equality at any given y_2. As y_2 increases, the left-hand side of the first equation decreases so $\hat{y}_1 + y_2$ must decrease. Therefore $\hat{y}_1(y_2)$ must decrease faster than y_2. Applying a similar argument to the other equation ensures any fixed point is unique. The next proposition shows the fixed point constrains the equilibrium bargain.

Lemma 4 (Dutta (2012), Proposition 3)**.** *Suppose $\hat{\sigma}$ is a pure strategy Markov perfect equilibrium of the game I have described in Proposition 1 under Assumption 1 but not necessarily 3 and 4. Suppose i and j could form a Pareto-improving contract in period t. Then their equilibrium demands are equal and are bounded above and below by $y_1 p_D^M + (1 - y_1)p_D^H$ and $y_2 p_D^H + (1 - y_2)p_D^M$, respectively, where y_1 and y_2 come from Lemma 3.*

Lemma 5. *For a given ij pair, let $y_{1,(n)}$, $y_{2,(n)}$ be the y_1, y_2 corresponding to Lemma 3 in game n for a given ij pair. (If ij does not have a strictly Pareto-improving pair, choose some $y_1, y_2 \in (0, 1)$ satisfying $y_1 + y_2 = 1$.) Under the conditions of Proposition 1, $y_{1,(n)} + y_{2,(n)} \to 1$ with a convergence rate that is uniform in (i, j).*

A.2 Proofs

Proof of Lemma 1. Intermediate value theorem applied to the continuous GFT functions.

□

Proof of Lemma 2. First, I show there is no concession in equilibrium. Suppose ij concede in equilibrium. If both sides concede, then one delegate could do better by improving their demand and this is not an equilibrium. Suppose one side concedes. Consider that side instead deviating at the demands stage to demand the contract reached. This does not change the demands by any other delegate. This deviation also does not change the expected profit for any other concession decision. Therefore since the strategies are Markov, all other concession decisions are unaffected. Therefore resulting firm profits are unaffected, but the delegate avoids the concession cost and the demand is strictly dominated. Therefore there is no concession in equilibrium.

Now I show by contradiction that there is never a subgame agreement reached by a hospital demanding a strictly lower price than the insurer they negotiate with. By Assumptions 1 and 2, both parties could strictly improve their profits by demanding the contract the other side demands. Contradiction. Therefore demands are equal in equilibrium of any pair that successfully reaches a contract.

□

Proof of Lemma 3. I am proceeding assuming there is at least one y_1 and y_2 such that $y_1 + y_2 > 1$ and:

$$V^H\left(y_2 p_D^H + (1-y_2)p_D^M\right) - V^H(p_D^H) = c^H\left((p_D^M - p_D^H)(y_1 + y_2 - 1)\right)$$
$$V^M\left(y_1 p_D^M + (1-y_1)p_D^H\right) - V^M(p_D^M) = c^M\left((p_D^M - p_D^H)(y_1 + y_2 - 1)\right).$$

Since $y_1 + y_2 > 1$ and c is increasing for values above 0, the right-hand side of both equations is positive. Therefore the left-hand side is positive, i.e. $y_1, y_2 < 1$.

Now consider more generally the function $\hat{y}_1(y_2) : [0,1] \to [0,1]$ to solve $V^H\left(y_2 p_D^H + (1-y_2)p_D^M\right) - V^H(p_D^H) = c^H\left((p_D^M - p_D^H)(\hat{y}_1(y_2) + y_2 - 1)\right)$, i.e:

$$\hat{y}_1(y_2) = \frac{(c^H)^{-1}\left(V^H\left(y_2 p_D^H + (1-y_2)p_D^M\right) - V^H(p_D^H)\right)}{p_D^M - p_D^H} + 1 - y_2$$

As pointed out by Dutta in the differentiable case, $\hat{y}_1$ is a continuous function, $\hat{y}_1(1) = 0$, $\hat{y}_1(0) > 1$, and $\hat{y}_1(y_2)$ decreases strictly faster than y_2 since an increase in y_2 by a unit and a decrease in y_1 by one unit leaves $c^H((p_D^M - p_D^H)(y_1 + y_2 - 1))$ unchanged but reduces $V^H(y_2 p_D^H + (1-y_2)p_D^M)$. The function $\hat{y}_2(y_1)$ has the same properties.

By the intermediate value theorem, there there is a fixed point to the function $\hat{y}_1(\hat{y}_2(y_1))$. Since increasing y_1 by ε increases $\hat{y}_1(\hat{y}_2(y_1))$ by strictly more than ε, that fixed point is unique. Since the fixed point (y_1^*, y_2^*) is in $(0,1)$, it must generate positive left-hand sides so that $y_1^* + y_2^* > 1$. □

Proof of Lemma 4. This claim is almost exactly Dutta (2012)'s Proposition 3. Suppose the hospital delegate demands price at least $z_1 p_D^M + (1-z_1)p_D^H$ and the insurer delegate demands price at most $z_2 p_D^H + (1-z_2)p_D^M$. Since there is no concession in equilibrium (Lemma 2), it must be $z_1 = (1-z_2)$, so that $z_1 + z_2 = 1$. $z_1 = 1$ and $z_2 = 0$ corresponds to the hospital getting all of the surplus, whereas $z_2 = 1$ and $z_1 = 0$ corresponds to the insurer getting all of the surplus.

After appropriate notation changes, the claim is almost in the setup of Dutta (2012). There is a change to concession costs if both concede, but since that requires bilateral deviation it is irrelevant to the equilibrium and the same result holds. $\square$

Proof of Lemma 5. I proceed for some ij and a sequence of games n satisfying $p_D^M > p_D^H$ and $y_1 + y_2 > 1$; the claim is immediate for the other n.

Recall that $V^H(p_D^M) - V^H(p_D^H), V^M(p_D^H) - V^M(p_D^M) \to_n 0$ by Assumption 3 and $V' \geq \varepsilon B > 0$ by Assumption 1, so that $p_D^M - p_D^H \to_n 0$.

Recall that $y_{1,(n)}$, $y_{2,(n)}$ are defined as the solution to:

$$V^H\left(y_{2,(n)}p_D^H + (1 - y_{2,(n)})p_D^M\right) - V^H(p_D^H) = c^H\left((p_D^M - p_D^H)(y_{1,(n)} + y_{2,(n)} - 1)\right)$$
$$V^M\left(y_{1,(n)}p_D^M + (1 - y_{1,(n)})p_D^H\right) - V^M(p_D^M) = c^M\left((p_D^M - p_D^H)(y_{1,(n)} + y_{2,(n)} - 1)\right).$$

Costs go to zero quickly enough that the infinite right-derivative at zero dominates. The cost functions must tend to zero because $p_D^M - p_D^H \to_n 0$ and $y_{1,(n)} + y_{2,(n)} - 1$ is bounded. The cost functions are also lower-bounded by a function with an infinite right-derivative (Assumption 3). As a result, there is a sequence of $\epsilon_n \to_n 0$ such that $c\left((p_D^M - p_D^H)(y_{1,(n)} + y_{2,(n)} - 1)\right) > B(p_D^M - p_D^H)(y_{1,(n)} + y_{2,(n)} - 1)/(2\epsilon_n)$ for all n large enough and all i, j with $y_{1,(n)} + y_{2,(n)} > 1$.

Note that by construction, $V^H\left(y_{2,(n)}p_D^H + (1 - y_{2,(n)})p_D^M\right) - V^H(p_D^H) \leq B(1 - y_{2,(n)})(p_D^M - p_D^H)$ and $V^M\left(y_{2,(n)}p_D^H + (1 - y_{2,(n)})p_D^M\right) - V^H(p_D^H) \leq B(1 - y_{1,(n)})(p_D^M - p_D^H)$. As a result:

$$B(p_D^M - p_D^H)(y_{1,(n)} + y_{2,(n)} - 1)/\epsilon_n < B(2 - y_{1,(n)} - y_{2,(n)})(p_D^M - p_D^H)$$
$$y_{1,(n)} + y_{2,(n)} - 1 < 2\epsilon_n \to_n 0.$$

Since $1 \leq y_{1,(n)} + y_{2,(n)} \leq 1 + o_n(\epsilon_n)$ for ϵ_n independent of i, j, this completes the proof.

$\square$