Which Bargaining Solutions are Identified under Unobserved Information Timing?

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Abstract

This paper studies identification of bargaining parameters under uncertainty. I show that Nash bargaining weights are not identified in general: for any two interior bargaining weights, there is a pair of games with different information timing under which the weights are observationally equivalent. In contrast, Kalai proportional bargaining admits a moment on expected gains from trade that remains valid whether bargaining occurs ex ante, ex post, or somewhere in between. I show that among bargaining solutions that satisfy independence of irrelevant alternatives, only Kalai proportional bargaining is identified in general: I show identification implies the social choice property of concavity, and I extend Myerson (1981)'s results to rule out other families. I apply these results to multiperiod contracting in transferable utility games, where Nash and Kalai coincide and my analysis can focus on the role of unobserved information on aggregate gains. I show that bargaining weights may be unidentified with multiperiod contracts, but identification can be restored under plausible econometric restrictions.

A common econometric strategy for structural analysis of bargained outcomes is as follows. First, the researcher conjectures a model of gains from trade. Second, the researcher constructs a moment based on how a family of bargaining solutions, typically Nash solutions, translate those gains to outcomes. Third, the researcher uses generalized method of moments to translate outcomes to bargaining weights.

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Within this framework, the researcher must take a stance on information. At one extreme, negotiation might occur with only the information available to the researcher. Pure ex ante bargaining is usually implausibly generous to the dataset at hand. At another extreme, negotiation might occur with full information on cost and demand shocks. Pure ex post bargaining is indefensible in a dynamic model, but at least in a static model, one might expect ex post bargaining over the realized Pareto frontier to imply similar behavior to ex ante bargaining over the expected frontier.

This paper shows that under nontransferable utility (NTU) Nash bargaining, the researcher's stance on information timing is substantive for GMM-implied bargaining weights. The outcome of Nash bargaining depends on both aggregate gains from trade and the slope of the Pareto frontier. As a result, when the slope of the Pareto frontier may be uncertain, the researcher's stance on the degree to which unobserved information is known at the time of bargaining will have empirical content for the predicted bargaining weight.

One might hope to recover Nash bargaining weights through a clever identification strategy, but I show no such approach can work in general: Nash bargaining weights may not be identified. For any bargaining weights $\tau_1, \tau_2 \in (0, 1)$, I provide a distribution of feasible utilities for which the outcome of choosing a price through Nash bargaining with weight τ_1 before learning the NTU Pareto frontier is observationally equivalent to the outcome of choosing a price through Nash bargaining with weight τ_2 after learning the Pareto frontier. As a result, even if the researcher knows all parameters other than the Nash bargaining weight, there may be no way to identify the negotiators' bargaining weight without also knowing the information set under which bargaining occurs.

I show that under Kalai proportional bargaining, bargaining weights can be recovered through a simple moment on expected gains from trade. Kalai is the solution concept that chooses the best agreement that splits gains from trade proportionally to fixed bargaining weights τ and $1 - \tau$. This solution concept agrees with Nash for transferable utility (TU) games, but possesses a robust moment whether bargaining is TU or NTU. Under ex ante bargaining, the ratio of expected gains from trade must be proportional to $\tau/(1-\tau)$. Under ex post bargaining, the ratio of realized gains from trade must be proportional to $\tau/(1-\tau)$. By iterated expectations, in both cases, the ratio of expected gains from trade remains $\tau/(1-\tau)$. In this way, the researcher can recover bargaining weights whether bargaining is conducted ex ante, ex post, or somewhere in between.

One might wonder whether other bargaining solutions have a similarly robust moment; I show that no other family of bargaining solutions satisfying independence of irrelevant alternatives (IIA) is even identified in general. The uniqueness result follows from Myerson (1981)'s characterization of concavity in more general social choice problems. The concavity property is that ex ante, no one should prefer to commit to negotiating ex post. Concavity is a desirable property for bargaining under uncertainty, because negotiating ex ante should allow the players to reallocate welfare across states of the world to achieve Pareto improvement. When applied to bargaining problems, Myerson's results show that only proportional and utilitarian bargaining solutions can satisfy IIA and be concave. I show that concavity is a necessary condition for identification, ruling out all other families, and that utilitarian solutions are not identified in general, leaving only the Kalai proportional solution.

The static results establish that Kalai proportional bargaining permits identification without knowledge of information timing, but unobserved information creates even stronger identification problems in dynamic models. I study the challenges created by multiperiod contracting with possible unobserved information on future states. To focus on the role of future information, I study the TU case in which all uncertainty is over aggregate gains from trade, and not marginal values. In this setting, Nash and Kalai coincide, so that the results also establish identification for multiperiod TU Nash bargaining.

I show that when contracts remain in place for multiple periods, Kalai proportional weights may be nonparametrically unidentified. With single-period contracts, Kalai weights can be identified from the static moment; the discount factor is irrelevant in the period-by-period model. However, when contracts remain in place for multiple periods, I can construct a game in which the forward-looking solution has a myopic equivalent with different bargaining weights. However, this argument does not apply for all multiperiod settings: if current choices are correlated with future paths that are uncorrelated with current utility shocks, then one can reject myopia. This analysis leverages results from the companion paper Dorn (2026), which establishes that one can apply the step-by-step property to obtain a much simpler equivalent representation of recursive Kalai bargaining.

I provide two sufficient conditions for identification with multiperiod contracts. First, there is identification if there are no unobserved components of utility and there is variation in agreements conditional on the observed states. In this case, the discounting factor can be identified from the association of future states and starting prices. Alternatively, there is identification from an instrument satisfying an appropriate exclusion restriction, in particular only affecting value functions through inflation expectations. In this second case, the discounting factor can be identified from the instrument's joint association with future inflation and starting prices

My IIA uniqueness result is essentially a gentle extension of Myerson (1981)'s work. My work on (non)identification relates to work on dynamic discrete choice models (Abbring, 2010). The first set of identification conditions are motivated by a remark by Rust (1994) that in these settings, some identifying power may be derived from "agents who make different choices in the same state," and the second set is motivated by exclusion conditions in Magnac and Thesmar (2002) and Abbring and Daljord (2020), but is specifically motivated by inflation expectations. In dynamic discrete choice problems, the first set of conditions is known to be insufficient for identification (Rust, 1994) and the second condition has no natural analog. This work also relates indirectly to theoretical work on recursive bargaining in bilateral settings (Sorger, 2006; Flamini, 2020; Dutta, 2021).

The remainder of this paper is as follows. Section 1 reviews the Kalai (1977) proportional and Nash (1950) bargaining solutions in a static context. Section 2 analyzes the identification of bargaining weights under possible uncertainty in a single-period environment. Section 3 analyzes the identification of bargaining weights and discount factors in an infinitely-lived but TU environment. Section 4 concludes.

1 Static Review

This section reviews the Kalai proportional and Nash bargaining solutions, and some key properties of those solutions in a single bargaining problem over a known Pareto frontier.

A Nash bargaining problem G (for game) is a closed set V of feasible agreement values in $\mathbb{R}^2$ and a disagreement value $v^D \in V$. I use "$\geq$" and "$>$" to refer to the pointwise comparison: $v \geq v'$ if and only if $v_1 \geq v_1'$ and $v_2 \geq v_2'$, and analogously $v > v'$ means $v_1 > v_1'$ and $v_2 > v_2'$. I refer to a set of problems (V, v^D) as a family $\mathcal{G}$, and will make a few regularity assumptions.

Definition 1 (Regularity conditions)**.** *A game* (V, v^D) *satisfies* convexity *if for all* $v_1, v_2 \in V$ *and* $\lambda \in [0,1]$*,* $\lambda v_1 + (1-\lambda)v_2 \in V$*. It satisfies* comprehensiveness *if* $v \in V$ *and* $v^D \leq v' \leq v$ *implies* $v' \in V$*. A family* $\mathcal{G}$ *of games* G *satisfies these properties if every game* $G \in \mathcal{G}$ *satisfies the property.*

Convexity is an important regularity condition for bargaining games (Shimer, 2006), and generally reduces a bargaining problem to a Pareto frontier and disagreement value. Comprehensiveness is sometimes called free disposal; comprehensiveness of the convex hull of the Pareto frontier and disagreement point ensure that the Kalai proportional solution is well-defined (Roth, 1979).

Within a bargaining game, there are many ways of choosing an allocation that makes both sides better off than disagreement. A particular bargaining solution is a function $f : \mathcal{G} \to \mathbb{R}^+$ such that $f(V, v^D) \in V$ and $f(V, v^D) \geq v^D$. I write that $f' = f$ on $\mathcal{G}$ if for all $(V, v^D) \in \mathcal{G}$, $f(V, v^D) = f'(V, v^D)$, and shorthand $f' = f$ if the two are equal on $\mathcal{G}$ when the family is clear from context.

Definition 2 (Kalai proportional bargaining)**.** *The* Kalai proportional bargaining solution *with player-1 weight* $\tau \in [0,1]$ *is* $f_\tau^{(Kalai)}(V, v^D) = \left(\sup v_1 : v \in V \cup \{v^D\}, (1-\tau)(v_1 - v_1^D) = \tau(v_2 - v_2^D), \sup v_2 : v \in V \cup \{v^D\}, (1-\tau)(v_1 - v_1^D) = \tau(v_2 - v_2^D)\right)$.

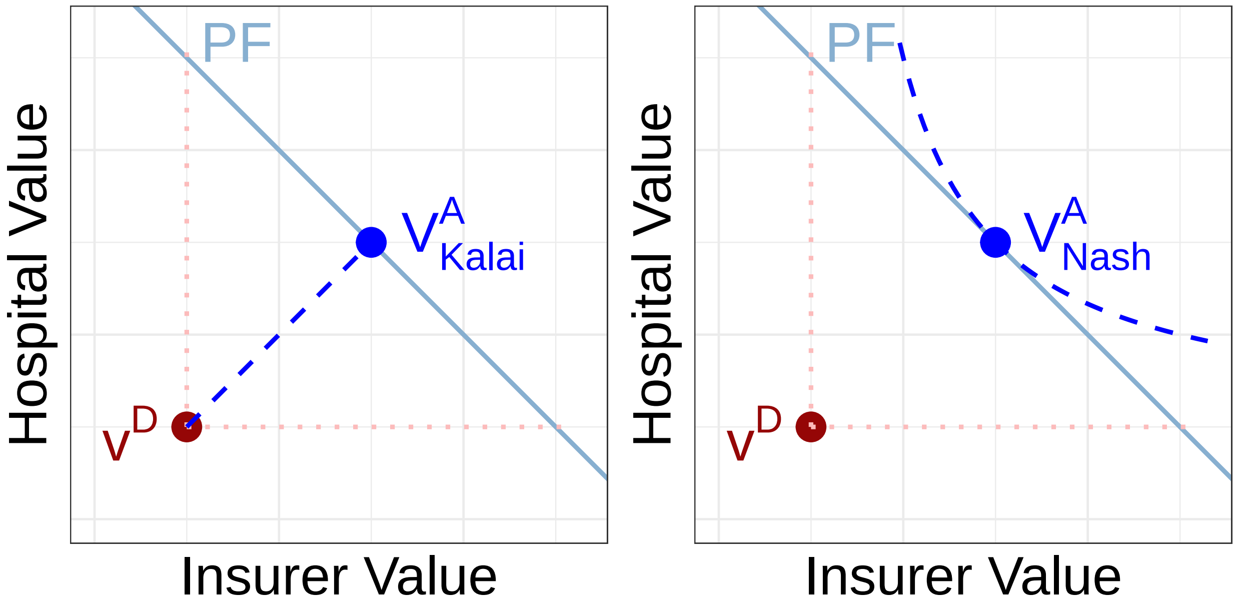

Figure 1: Left: the Kalai proportional bargaining solution for bargaining between an insurer (x axis) and hospital (y axis) with Pareto frontier PF in light blue and disagreement value v^D in red, and Pareto constrained indicated by dotted lines. The desired-surplus-split ray $v = v^D + x(1-\tau)/\tau$ is indicated by the dashed blue ray, and the resulting agreement is indicated by v^A_{Kalai}. Right: the Nash bargaining solution to the same game. The dashed line indicates the Nash bargaining indifference curve, and the resulting agreement is indicated by v^A_{Nash}.

The Kalai proportional solution for bargaining between a hospital (player 2) and insurer (player 1) is illustrated in Figure 1(a). Draw a dashed ray to indicate the values $v \in V$ that yield positive gains from trade and the desired surplus split $\tau(v_2 - v_2^D) = (1-\tau)(v_1 - v_1^D)$. The solution v^A_{Kalai} is the best agreement along this desired-surplus-split ray.

Definition 3 (Nash bargaining)**.** *The Nash bargaining solution with player-one bargaining weight* $\tau \in [0,1]$ *is* $f_\tau^{(Nash)}(V, v^D) = \text{argmax}_{v \in V} \left(v_1 - v_1^D\right)^\tau (v_2 - v_2^D)^{1-\tau}$.

The Nash bargaining solution is illustrated in Figure 1(b). The Nash bargaining solution can be viewed as maximizing a joint utility function, with player one receiving greater weight.

The joint utility function generates a joint indifference curve. In well-behaved problems with negotiations over a price, the Nash bargaining first-order condition at the agreed price p^*_{Nash} can be rewritten as:

$$\tau v_1'(p^*_{\text{Nash}})(v_2^* - v_2^D) = -(1-\tau)v_2'(p^*_{\text{Nash}})(v_1^* - v_1^D).$$

Kalai and Nash bargaining will have the same predictions when bargaining is TU: if the Pareto frontier is a line segment with a constant slope of negative one.

Definition 4 (Transferable utility)**.** *A family of games $\mathcal{G}$ satisfies* transferable utility *(TU) if for every $(V, v^D) \in \mathcal{G}$, there is a $\lambda > 0$ such that the set of $v \in V$ with $v \geq v^D$ is the convex hull of v^D, $v^D + (\lambda, 0)$, and $v^D + (0, \lambda)$.*

Equivalence follows by algebra.

Lemma 1 (Nash and Kalai coincide for TU games)**.** *Suppose $\mathcal{G}$ satisfies TU. Then $f_\tau^{(Nash)} = f_\tau^{(Kalai)}$ on $\mathcal{G}$.*

Proof. See Appendix A.2 for the short proof. □

Some important advantages of Nash bargaining are that it is weakly Pareto-optimal and satisfies independence of irrelevant alternatives and scale-invariance (Serrano, 2005).

Definition 5 (Bargaining properties)**.** *For $\alpha \in \mathbb{R}^d$ for $d = 1$ or 2, let $\alpha V = \{(\alpha_1 v_1, \alpha_2 v_2) : v \in V\}$. Also let $\mathcal{G}$ be the set of games that are convex and comprehensive. A bargaining solution f is (i)* weakly Pareto-optimal *(WPO) if for all $(V, v^D) \in \mathcal{G}$, there is no $v \in V$ such that $v > f(V, v^D)$; (ii) satisfies* independence of irrelevant alternatives *(IIA) if $(W, v^D), (V, v^D) \in \mathcal{G}$, $W \subseteq V$, and $f(V) \in W$ implies $f(W, v^D) = f(V, v^D)$; and (iii) is* scale-invariant *if for all diagonal 2-by-2 matrices Σ with positive diagonal elements and all $(V, v^D) \in \mathcal{G}$, $f(\Sigma V, \Sigma v^D) = \Sigma f(V, v^D)$.*

WPO is a minimal condition for a bargaining solution to be plausible. IIA is a necessary condition for f to maximize some notion of utility (Myerson, 1981).[1] I slightly abuse notation and write that if f satisfies IIA, $(V, v^D) \in \mathcal{G}$, and $f(V, v^D) \in W \subseteq V$, then $f(W, v^D) = f(V, v^D)$ even if W is not comprehensive. Scale invariance implies that if one modifies the scale of one side's gains from trade by a fixed constant, then the other side's outcome is unaffected.

Two important advantages of Kalai proportional bargaining are that it satisfies concavity and step-by-step.[2]

Definition 6 (Concavity and step-by-step)**.** *For bargaining games $G = (V, v^D)$ and $G' = (W, u^D)$ and $\lambda \in [0, 1]$, define $\lambda G + (1-\lambda)G' = \big(\lambda V + (1-\lambda)W, \lambda v^D + (1-\lambda)u^D\big)$. A bargaining solution f satisfies concavity if for all G, G' such that $\{G, G'\}$ satisfies convexity and comprehensiveness and all $\lambda \in [0, 1]$, $f(\lambda G + (1-\lambda)G') \geq \lambda f(G) + (1-\lambda)f(G')$. A bargaining solution f satisfies step-by-step for $G = (V, v^D)$ if for all sets W with $f(W, v^D) \in V$, $f(V, f(W, v^D)) = f(V, v^D)$.*

The concavity property is that ex ante bargaining over expected values before the Pareto frontier is realized should produce weakly better outcomes for all players than the expected value of ex post bargaining after the Pareto frontier is realized. The step-by-step property is that moving the value of disagreement upwards based on a first-step agreement should not affect the resulting second-step outcome. Dorn (2026) shows that step-by-step allows for simplifying recursive bargaining problems. This paper shows that concavity is a necessary property for information-robust identification.

Kalai proportional bargaining is not scale invariant, so it can only coincide with Nash bargaining within a family of games like TU that implicitly define a scale of relative utility. The value of scale invariance is more subtle than it seems at first glance: scaling player

[1] The IIA axiom can be replaced by lower-level conditions that are outside the scope of this work (Thomson and Myerson, 1980).

[2] Myerson (1981) defines concavity for more general social choice problems which lack a notion of disagreement. Bargaining solutions are a subset of social choice functions applied to a subset of social choice problems, so the concept can be extended to bargaining games.

2's utility by $\lambda > 0$ and the Kalai proportional bargaining weight ratio $\tau/(1-\tau)$ by λ^{-1} yields the same implied outcomes from the perspective of player 1, and evidence from the lab suggests the negotiators can be scale-varying in monetary games (Nydegger and Owen, 1974; Duffy et al., 2021). Instead, the most impactful consequence from losing scale-invariance that I have found is that the bargaining solution cannot be microfounded in terms of von Neumann-Morgenstern utility alone.[3]

I now move to dynamic games, where I make three key assumptions. First, I assume that information is shared; bargaining under asymmetric information is a challenging frontier topic (Larsen and Zhang, 2021; Chen et al., 2024). Second, I assume that expectations are rational; bargaining under irrational expectations would make GMM biased even if bargaining was known to occur ex ante. Third, I assume that the players apply a single fixed per-period discount factor $\beta \in [0, 1)$; alternative shared time preferences could be accommodated at the cost of additional notation, but asymmetric time preferences would imply a perverse Pareto improvement where the patient player offers the myopic player a disproportionate payment today in exchange for a sufficiently larger payment tomorrow.[4]

2 Single-Period Identification in NTU Games

2.1 Nash GMM Bias and Possible Nonidentification

Many empirical dynamic models include the possibility of uncertainty at the time of contracting. It is therefore often desirable to analyze a model in which agents have access to some, but potentially not all, information beyond the dataset at hand. It turns out that when the researcher wishes to be agnostic about the information set available at the time of

[3] I present a microfoundation of the Nash-in-Kalai model in the companion note **?**. The microfoundation requires some device that fixes relative utility scales: I follow Dutta (2012) and use revocation costs. This is the first microfoundation for either Nash-in-Kalai or Nash-in-Nash that allows the NTU case in which the two disagree.

[4] That said, such perverse examples may occur in practice. For example, children have been known to offer generous initial terms in exchange for disproportionate future computer access (Dorn and Dorn, 2001). Modeling such behavior is outside the scope of this work.

bargaining, there may not be a way to identify Nash bargaining weights under NTU.

It is easiest to quantify the role of uncertainty in econometric modeling in a single-period game. Consider the following class of games parameterized by $\big(EA \in \{0,1\}, u = \{u_1(p \mid x), u_2(p \mid x)\}, v^D\big)$:

1. If $EA = 1$ (for ex ante), the players ($i = 1, 2$) have the opportunity to negotiate over a price $p \in \mathbb{R}$ using bargaining solution f.
2. The game type X is drawn from $Unif(0, 1)$ and revealed to the players.
3. If $EA = 0$, the players have the opportunity to negotiate over a price $p \in \mathbb{R}$.
4. If the sides have an agreement ($A = 1$), then player i receives utility $u_i(p \mid X) > 0$. If the sides have no agreement ($A = 0$), then both players receive v^D.
5. The researcher observes the distribution P, the game type X, the utility functions u_1 and u_2, the disagreement payoff v^D, and the price p^*.

For example, when $EA = 1$, the sides may negotiate over a price whose effect on profits depends on uncertain demand shocks.

It would be convenient for empirical practice to have a moment of the form $E[g(p, X; \tau)] = 0$ to infer Nash bargaining weights. One type of moment might involve marginal costs (e.g. Gowrisankaran et al., 2015), which corresponds here to marginal utility. The researcher may then leverage the ex-ante or ex-post moments on putative bargaining weights $\bar{\tau}$:

$$m_{EA}(\bar{\tau}) = E\left[u_2'(p \mid X) - \frac{\bar{\tau}}{1-\bar{\tau}} \frac{E[u_2(p \mid X) - v_2^D]E[-u_1'(p \mid X)]}{E[u_1(p \mid X) - v_1^D]}\right]$$

$$m_{EP}(\bar{\tau}) = E\left[u_2'(p \mid X) - \frac{\bar{\tau}}{1-\bar{\tau}} \frac{\left(u_2(p \mid X) - v_2^D\right)\left(-u_1'(p \mid X)\right)}{u_1(p \mid X) - v_1^D}\right].$$

The validity of these moments will depend on getting the timing of information right. If bargaining occurs ex ante before uncertainty over the shape of an NTU Pareto frontier is resolved, the players' expected gains from trade depend on making comparisons of the relative scales of their feasible utility across potential states of the world, so that affine transformation of one player's utility in some states can generate a different outcome for the other player. In contrast, if bargaining occurs ex post after the shape of the Pareto frontier is realized, the scale-invariance property of Nash bargaining ensures that the Nash solution is invariant to such affine transformation. This creates ambiguity from the perspective of the researcher that does not observe information sets: outcomes that are more favorable to player 1 could reflect a better bargaining weight, or they may reflect information on utility transformations being revealed in a way that systematically helps that player. As a result, bargaining weights estimated based on an incorrect conjecture of information timing can be systematically biased.

For an example of how unobserved information sets on utility nontransferability can introduce bias, consider games for which the observable state X parameterizes pre-transfer gains π_i and the marginal utility of price $\delta_i > 0$. With staggered contracts, a negotiated bilateral price may have internalized contracting externalities on other prices that are asymmetric, so that δ_1 may not equal δ_2. The precise conditions under which there is bias are somewhat subtle due to the nonlinearity of the problem, but in essence, the researcher's stance on information timing will introduce bias if there is an asymmetric correlation between levels of gains from trade (π) and the slope of the Pareto frontier ($-\delta_2/\delta_1$).

Proposition 1 (Absence of a robust Nash moment condition)**.** *Consider the outcome of bargaining games with $EA = 0$, the bargaining solution $f_\tau^{(Nash)}$ for some $\tau \in (0, 1)$, and utility $u_1(p \mid X) = \pi_1 - \delta_1 p$, $u_2(p \mid X) = \pi_2 + \delta_2 p$, and $v^D = 0$ for some random $\pi_i \in \mathbb{R}, \delta_i > 0$ with appropriate moment conditions. Suppose $\frac{E[\delta_1]}{E[\delta_2]} E\left[\pi_1 \left(\frac{\delta_2}{\delta_1} - \frac{E[\delta_2]}{E[\delta_1]}\right)\right] \neq E\left[\pi_2 \left(\frac{\delta_1}{\delta_2} - \frac{E[\delta_1]}{E[\delta_2]}\right)\right]$. Then $m_{EP}(\tau) = 0$, but $m_{EA}(\tau) \neq 0$.*

Proof. By construction, $m_{EP}(\tau) = 0$. $m_{EA}(\tau)$ is well-defined under appropriate moment conditions. Note that $p^*_{EP} = (1-\tau)\frac{\pi_1}{\delta_1} - \tau\frac{\pi_2}{\delta_2}$. Then:

$$
\begin{aligned}
m_{EP}(\tau) &= E\left[\delta_2 - \frac{\tau}{1-\tau}\frac{E\left[(1-\tau)(\pi_2+\pi_1(\delta_2/\delta_1)\right]E\left[\delta_1\right]}{\tau E\left[\pi_1+\pi_2(\delta_1/\delta_2)\right]}\right] = E\left[\delta_2 - \frac{E\left[\pi_1\frac{\delta_2}{\delta_1}+\pi_2\right]E\left[\delta_1\right]}{E\left[\pi_1+\pi_2(\delta_1/\delta_2)\right]}\right] \\
&= E\left[\delta_2 - \frac{E[\pi_2]E[\delta_1]+E[\pi_1]E[\delta_2]+E\left[\pi_1\left(\frac{\delta_2}{\delta_1}-\frac{E[\delta_2]}{E[\delta_1]}\right)\right]E\left[\delta_1\right]}{E[\pi_1]+E[\pi_2]\frac{E[\delta_1]}{E[\delta_2]}+E\left[\pi_2\left(\frac{\delta_1}{\delta_2}-\frac{E[\delta_1]}{E[\delta_2]}\right)\right]}\right] \\
&\neq E\left[\delta_2 - \frac{E[\pi_2]E[\delta_1]E[\delta_2]/E[\delta_2]+E[\pi_1]E[\delta_2]+E[\delta_2]E\left[\pi_2\left(\frac{\delta_1}{\delta_2}-\frac{E[\delta_1]}{E[\delta_2]}\right)\right]}{E[\pi_1]+E[\pi_2]\frac{E[\delta_1]}{E[\delta_2]}+E\left[\pi_2\left(\frac{\delta_1}{\delta_2}-\frac{E[\delta_1]}{E[\delta_2]}\right)\right]}\right] = 0.
\end{aligned}
$$

□

The term $E\left[\pi_1\left(\frac{\delta_2}{\delta_1}-\frac{E[\delta_2]}{E[\delta_1]}\right)\right]$ roughly captures the correlation of player one's pre-transfer gains with the realized Pareto frontier slope δ_2/δ_1. If such a correlation is large for one player and not the other, then a moment based on the implied ex-ante marginal costs will be biased if the prices are negotiated ex-post, regardless of the (interior) bargaining weight. A similar result can also be derived if prices are negotiated ex-ante, but some care must be taken to ensure $m_{EP}(\tau)$ is well-defined.

Proposition 1 shows that under dynamic NTU Nash bargaining, a proposed moment's validity will often hinge on getting the structure information correct. This may just be an inconvenience: there can be a more clever strategy to recover identification. However, it turns out that no such clever identification strategy can exist in general.

The rest of this section focuses on a minimal condition for deriving a robust moment for estimation of bargaining parameters: whether a bargaining solution can generally be identified in these single-period games. I try to adapt notation from Magnac and Thesmar (2002)'s analysis of identification of dynamic discrete choice problems. In this section, a structure b is a value of EA, a distribution P, utility functions u, and strategies σ such that σ is the output of applying a bargaining solution f to the generated bargaining problem. I write that structures b and b' are observationally equivalent if they generate the same observables:

u_1, u_2, P and the joint distribution of (X, p^*). I shorthand observational equivalence as $b \Leftrightarrow b'$. I write that bargaining solutions f and f' are observationally equivalent if for every b with the bargaining solution f, there is a b' with the same EA, P, u, the bargaining solution f', and some σ' such that $b' \Leftrightarrow b$. In this case, I write $f \Leftrightarrow f'$. I abuse notation and write $f(b)$ for the set of feasible outcomes after replacing the bargaining solution in b with f, and write $f(b) \Leftrightarrow f'(b')$ if the intersection of $f(b)$ and $f'(b')$ is nonempty.

Definition 7 (Single-period identification). *Let $\mathcal{F}$ be a family of bargaining solutions. I write that $\mathcal{F}$ is* single-period identified *if $b \Leftrightarrow b'$ implies $f(b) \Leftrightarrow f(b')$ for structures b, b' such that the convex hull of $\{(u_1(p \mid X), u_2(p \mid X))\} \cup \{0\}$ is comprehensive. I write that $\mathcal{F}$ is* single-period unidentified *if $\mathcal{F}$ is not single-period identified.*

If $\mathcal{F}$ is single-period unidentified, then a researcher who mistakenly conjectures that bargaining is ex post ($EA = 0$) can infer a bargaining solution that has the wrong counterfactual prediction even with a known information structure.

For these single-period games, it is possible to construct a game for which incorrectly conjecturing EA yields exact predictions, but an arbitrarily wrong estimate of the Nash bargaining weight τ.

Proposition 2 (Nash bargaining weights may not be identified under NTU). *Let $\mathcal{F}$ be a family of bargaining solutions that include $f_\tau^{(Nash)}$ and $f_{\tau'}^{(Nash)}$ for some distinct $\tau, \tau' \in (0, 1)$. Then $\mathcal{F}$ is single-period unidentified.*

Proof. Without loss of generality assume $\tau > \tau'$. Consider the utility functions from Proposition 1 with $\pi_1 = 1$, $\delta_1 = X^{-\lambda}/\tau$, $\pi_2 = (1 - \tau)X^\lambda$, and $\delta_2 = 1$ for $\lambda = \sqrt{\frac{\tau - \tau'}{\tau(1-\tau')}}$, and either (i) the structure b with $EA = 0$ and $f = f_\tau^{(Nash)}$, (ii) the structure b' with $EA = 1$ and $f = f_{\tau'}^{(Nash)}$, or (iii) the structure b'' with $EA = 1$ and $f = f_\tau^{(Nash)}$.

By the scale-invariance of Nash bargaining, the structure b generates a constant price of zero. By construction of λ, the structure b' generates a constant price of zero. Therefore $b \Leftrightarrow b'$. But by inspection, $b' \not\Leftrightarrow b''$. Therefore $\mathcal{F}$ is single-period unidentified. □

The result shows that there is no clever combination of information, much less a moment, that can suffice to identify Nash bargaining weights in all cases. In more complicated games with multiple periods and interactions, constructing a valid moment to recover bargaining parameters is even more difficult.

The proof of Proposition 2 only uses the scale-invariance property of Nash bargaining: intuitively, bargaining before the realization of uncertainty requires comparing utility scales across states of the world. As a result, Proposition 2 establishes that there cannot be a robust moment for any family of bargaining solutions that contains two scale-invariant solutions with distinct interior predictions for TU games.

2.2 Kalai's Information-Robust Moment

Under Kalai proportional bargaining, there is a simple moment on payments that is robust to the structure of information.

Lemma 2 (Existence of a robust Kalai proportional moment)**.** *Consider a game* $\big(EA \in \{0,1\}, u, v^D\big)$ *such that with probability one, the convex hull of* $\{(u_1(p \mid X), u_2(p \mid X)\} \cup v^D$ *is comprehensive. Let the realized price under* f_τ^{Kalai} *be* p^*_{EA}*. Then the moment* $E\big[\tau(u_2(p^*_{EA} \mid X) - v_2^D) - (1-\tau)(u_1(p^*_{EA} \mid X) - v_1^D)\big] = 0$ *holds regardless of information timing.*

Proof. The comprehensiveness requirement ensures the Kalai proportional solution is well-defined across values of EA. For games with $EA = 1$, the claim is immediate. For games with $EA = 0$, the realized price satisfies $\tau(u_2(p^*_0 \mid X)) = (1-\tau)(u_1(p^*_0 \mid X))$ almost surely, so that the moment holds by iterated expectations. □

Proposition 2 and Lemma 2 establish a trade-off. On the one hand, scale-invariant bargaining solutions like Nash bargaining may be theoretically advantageous. However, they cannot provide a moment on observable quantities that is robust to the structure of information. The Kalai proportional solution is not scale-invariant, but it provides such a robust moment.

One might wonder what other bargaining solutions also possess a robust moment for identification. It turns out that no other family of bargaining solutions that satisfy IIA can provide a moment that is robust to the structure of information. The proof uses arguments from Myerson (1981). In particular, I make the following restriction.

Assumption 1 (IIA and span)**.** $\mathcal{F}$ is a family of bargaining solutions that satisfy IIA; are distinct in the sense that no separate $f, f' \in \mathcal{F}$ satisfy $f = f'$ for all G generated by single-period games; and span all Pareto-efficient outcomes in the sense that if v is Pareto optimal in some V, then for every $v^D < v$, there is an $f \in \mathcal{F}$ such that $f(V, v^D) = v$.

The distinctness assumption ensures that $f \Leftrightarrow f'$ only if the two solutions are the same, and is required for $\mathcal{F}$ to be single-period identified. The span assumption ensures the researcher can always infer a bargaining weight from a Pareto-efficient agreement; a family that fails the span assumption is not amenable to empirical use.

Theorem 1 (IIA and single-period identification implies Kalai proportional bargaining)**.** *Suppose $\mathcal{F}$ is a family of bargaining solutions satisfying Assumption 1. Then $\mathcal{F}$ is single-period identified if and only if $\mathcal{F}$ is the family of Kalai proportional bargaining solutions.*

This result shows that no other family of bargaining solutions satisfying IIA can permit identification via a moment across information timing: any alternative family satisfying IIA would not even be identified.

The proof rests on several distinct claims, some of which may be of independent interest. The proof proceeds in three steps: first, that identification requires concavity; second, that only utiltarian or proportional solutions satisfy IIA and concavity; and third, utilitarian solutions are not identified. I reach the full proof on Page 16.

Concavity is necessary for identification.

Lemma 3 (Single-period identification implies concavity)**.** *Suppose $\mathcal{F}$ is a family of bargaining solutions satisfying Assumption 1, and such that there is an $f \in \mathcal{F}$ that is not concave. Then $\mathcal{F}$ is single-period unidentified.*

Proof. At a high level, suppose there is an f, G, G' such that player two strictly prefers the ex post outcome: $\lambda f + 2(G) + (1-\lambda) f_2(G') > f_2(\lambda G + (1-\lambda) G')$. By WPO, player one must weakly prefer the ex ante outcome. By IIA and span (see Appendix A.2 for details), there is a related game for which the ex post outcome under f is observationally equivalent to the ex ante outcome under some $f' \in \mathcal{F}$ that is more favorable to player two. □

Concavity is unique to two classes of bargaining solutions: utilitarian bargaining and proportional bargaining.

Lemma 4 (Concavity and IIA implies utilitarian or proportional)**.** *Suppose f satisfies WPO, IIA, and concavity. Then either (i) $f \Leftrightarrow f_\tau^{\text{Kalai}}$ for some $\tau \in [0,1]$, or (ii) f is utilitarian in the sense that there is a $\tau \in [0,1]$ such that $f(V, v^D) \in \operatorname{argmax}_{v \in V : v \geq v^D} \tau(v_1 - v_1^D) + (1 - \tau)(v_2 - v_2^D)$ for all V, v^D.*

Proof. The claim is a gentle extension of Myerson (1981)'s Theorem 2, but requires care to handle disagreement constraints. The details are in Appendix A.2. □

Thomson (1994) provides more background on utilitarian bargaining. A simple property is that it cannot identify bargaining weights in single-period games.

Lemma 5 (Single-period identification implies not utilitarian)**.** *Suppose for $\tau \in (0,1), \tau' \in [0,1]/\tau$, there are $f_\tau^{(Util)}, f_{\tau'}^{(Util)} \in \mathcal{F}$ with $f_\alpha^{(Util)}(V, v^D) \in \operatorname{argmax}_{v \in V : v \geq v^D} \alpha(v_1 - v_1^D) + (1 - \alpha)(v_2 - v_2^D)$ for all V, v^D. Then $\mathcal{F}$ is single-period unidentified.*

Proof. Without loss of generality assume that $\tau, \tau' < 1$. Write $f = f_\tau^{(Util)}$ and $f' = f_{\tau'}^{(Util)}$. Consider the games b and b' with $EA = 0$ and f and f', respectively, applied to $u_1(p \mid X) = 1 - p$ and $u_2(p \mid X) = \frac{\min\{\tau,\tau'\}/2}{1 - \min\{\tau,\tau'\}/2} p$. Then $p^* = 0$ under both games, so $b \Leftrightarrow b'$. Now let b'', b''' be f and f', respectively, applied to $u_1(p \mid X) = 1 - p$ and $u_2(p \mid X) = \frac{(\tau+\tau')/2}{1-(\tau+\tau')/2} p$. Then $b'' \not\Leftrightarrow b'''$, so $f \not\Leftrightarrow f'$ and $\mathcal{F}$ is single-period unidentified. □

I am now ready to prove Theorem 1.

Proof of Theorem 1. Direction 1: if $\mathcal{F}$ is single-period identified, then it is the family of Kalai proportional solutions. Proof by contradiction. By Lemma 3, if $\mathcal{F}$ is single-period identified, then each element is concave. By Lemma 4 and IIA, each element is utilitarian and/or proportional. By Lemma 5, there is an $m > 0$ such that the game $EA = 0$, $u_1(p) = mp$, $u_2(p) = 1 - p$ has $p^* \in \{0, 1\}$ for all solutions in $\mathcal{F}$ that are not proportional. Then by the span assumption, for all $\tau \in (0, 1)$, $f_\tau^{(Kalai)} \in \mathcal{F}$. Now suppose $f \in \mathcal{F}$ is utilitarian and not proportional. By single-period identification and $f_\tau^{(Kalai)} \in \mathcal{F}$ for interior τ, if $f \in \mathcal{F}$ is not proportional, f must predict gains-from-trade shares of either one or zero always. But then since f is utilitarian, f is also proportional. Therefore $\mathcal{F}$ only includes proportional solutions, and by the span assumption, $\mathcal{F}$ is the family of proportional solutions.

Direction 2: if every $f \in \mathcal{F}$ is proportional, then f is single-period identified. Let data be generated as in Section 2.1 from some $f \in \mathcal{F}$ with bargaining weight τ. Let $\hat{\tau}$ solve $\hat{\tau}E[u_2(p^* \mid X)] = (1 - \hat{\tau})E[u_1(p^* \mid X)]$. By WPO and the existence of a p generating strict gains from trade, at least one of $E[u_1(p^* \mid X)]$ and $E[u_2(p^* \mid X)]$ must be strictly positive. Therefore $\hat{\tau}$ is unique. By Lemma 2, $\hat{\tau} = \tau$. It remains to show that no other bargaining solution in $\mathcal{F}$ is observationally equivalent. Suppose $f' \in \mathcal{F}/f$. Then by Lemma 2, f' generates a different moment. Therefore $f \not\Leftrightarrow f'$. □

Having established that Kalai proportional bargaining provides an information-robust moment in a single-period world with NTU, I now study identification in the context of multiperiod contracting. I focus on TU for tractability and because TU is the dominant empirical case for dynamic bargaining.

3 Multiperiod Identification

Consider the following class of infinitely-lived TU bargaining games. Period t is as follows:

1. Players 1 and 2 learn their period utility shocks $\varepsilon_t = (\varepsilon_{t1}, \varepsilon_{2t})$, observable state x_t in a discrete space $\mathcal{X}$ with n distinct values in the support, unobservable information i_t,

and inflation factor $\phi_t > -1$. I write $h_t = (\varepsilon_t, x_t, i_t, \phi_t)$.

2. If the last period ended with a contract in place ($\ell_{t-1} > 1$), then the transfer is $Pay_t = (1 + \phi_t)Pay_{t-1}$, the remaining length is $\ell_t = \ell_{t-1} - 1$, and I write $R_t = 0$ for no new negotiation.

3. If last period ended with an expiring or expired contract ($\ell_{t-1} \leq 1$), then the players can bargain by mutual assent and choose a starting transfer Pay_t for a T-period agreement (so that $\ell_t = T$ and $R_t = 1$) or can disagree (so that $Pay_t = 0$, $\ell_t = 0$, and $R_t = 0$).

4. *Utility and transition.* Player i gets flow utility $1\{\ell_t > 0\}(u_i(x_t) + \varepsilon_{t,i} + Pay_t(2i - 3) - r_i R_t)$, where r_i is player i's new-contract negotiating cost. The new state h_{t+1} is drawn from some fixed distribution $P(h' \mid h)$ and the researcher observes (Pay, ℓ, x).

I implicitly impose that h follows a Markov process. The convention in the final step is that Pay_t is the payment from player 2 to player 1, but Pay_t is allowed to be either positive or negative.

I proceed assuming certain regularity conditions.

Assumption 2 (Magnac and Thesmar (2002))**.** $E[\varepsilon_{t,i} \mid x] = 0$, the agents calculate value using the true law of motion $P(h' \mid h)$. Further, $P(\varepsilon', x', \phi', i' \mid h) = P(\varepsilon' \mid x, \phi)P(x', \phi', i' \mid x, \phi, i)$.

These are essentially the regularity conditions maintained by Magnac and Thesmar, but allowing the possibility of unobserved information about future states.

For simplicity, I follow Collard-Wexler et al. (2019) and focus on games in which agreements are always formed. Disagreement based on ε introduces a selection bias that can be corrected with distributional knowledge, but which is outside the scope of this work. Even in the case of $T = 1$ and no unobserved utility, it is impossible to separately identify the levels of $u_i(x)$ from r_i: one can increase u_i and r_i to achieve the same real outcome. With

T-period agreements, it turns out that one can only hope to identify

$$\tilde{u}_i(x) \equiv u_i(x) - r_i / \sum_{t=1}^{T} \beta^{t-1}. \tag{1}$$

Similarly, even without unobserved utility, payment data cannot separately identify the components of

$$\tilde{Pay}(x) \equiv -\tau\tilde{u}_1(x) + (1-\tau)\tilde{u}_2(x). \tag{2}$$

I implicitly assume that the researcher has access to instruments that can identify bargaining weights from $\tilde{Pay}$. In the single-period Section 2.2, the realized utility functions would be valid instruments.

3.1 The Multiperiod Kalai Model is Only Identified with Single-Period Agreements

A structure b is a set of current-period effective utility functions $\tilde{u}_i(x)$, information i, next-period value functions $V_i(Pay, h, \ell)$ for $0 \leq \ell' \leq T$ and $i = 1, 2$, discounting factor $\beta \in (0, 1)$, and new-contract choice functions $d^*(h) = (\ell^*(h), Pay^*(h))^T$ such that if $\ell > 0$,

$$V_i(Pay, h, \ell) = 1\{\ell > 0\}\left(u_i(x) + \varepsilon_{t,i} + (2i-3)Pay\right) + \beta E[V_i(\phi' Pay', h', \ell - 1) \mid h],$$
$$V_i(Pay^*(h), h, 0) = \max\left\{V\left(Pay^*(h), h, \ell^*(h)\right) - r, \quad \beta E[V_i(0, h', 0) \mid h]\right\}.$$

A structure b is a *recursive Kalai solution* with bargaining weight τ if $V_i\left(Pay^*(h), h, \ell^*(h)\right) - r_i$ always reflects player i's outcome from applying the Kalai proportional bargaining solution to Nash's bargaining problem for Pareto frontiers of feasible $(V_1 - r_1, V_2 - r_2)$, disagreement point $\beta E[V_i(Pay, h', 0) \mid h]$, and player-one bargaining weight τ.

Definition 8 (Multiperiod identification)**.** *I write that two structures* b, b' *are* observationally equivalent *if* $P(Pay', \ell', x', \phi' \mid Pay, \ell, x, \phi)$ *are the same under* b *and* b'*, in which case I write* $b \Leftrightarrow b'$*. For a family of structures* B*, I write that* traditional bargaining parameters are identified *under* B *if* $b \Leftrightarrow b'$ *implies that* $\tilde{Pay}(x)$ *are the same functions under* b *and* b'*. If traditional bargaining parameters are not identified, I write that* B *is* unidentified*. I write that* β is identified *under* B *if* $b \Leftrightarrow b'$ *implies that* β *is the same under* b *and* b'*. If traditional bargaining parameters are identified under* B*, I write that* all bargaining parameters are identified *under* B *if* β *is also identified and I write that* only β is not identified *under* B *if* β *is not identified.*

With single-period Kalai proportional agreements, traditional bargaining parameters (but not the discounting factor β) are identified.

Proposition 3 (Identification of traditional bargaining parameters with single-period contracts)**.** *Let* $B^{(1)}$ *be the set of structures generated by recursive Kalai bargaining satisfying Assumption 2 with* $T = 1$*, and such that* $\ell^*(h) = 1$ *for all* (h) *in the support of the game. Then only* β *is not identified under* $B^{(1)}$*.*

Proof. First, I show that traditional bargaining parameters are identified. By Assumption 2, agreement and disagreement have the same next-period expected value. For any given τ, agreements Pay_t^* must satisfy

$$\begin{aligned} 0 &= -\tau\left(u_2(x) + \varepsilon_2 + Pay_t - r_2\right) + (1-\tau)\left(\tilde{u}_1(x) + \varepsilon_1 - Pay_t^* - r_1\right) \\ &= \tilde{Pay}(x) - \tau\varepsilon_2 + (1-\tau)\varepsilon_1 - Pay_t^* \\ E[Pay_t^* \mid x] &= \tilde{Pay}(x) + E\left[-\tau\varepsilon_2 + (1-\tau)\varepsilon_1 \mid x\right] = \tilde{Pay}(x), \end{aligned}$$

with the final line following by Assumption 2. Therefore traditional bargaining parameters are identified.

Next, I show that β is not identified. Take $u_1(x) = u_2(x) = 1$, $r = 1/2$, and $\varepsilon = 0$ constant. Let b be the Kalai proportional solution with $\tau = 1/2$ and $\beta = 0$ and let b' be the solution with $\tau = 1/2$ and $\beta = 1/2$. By inspection, $b \Leftrightarrow b'$. Therefore β is not identified. □

While the discount factor β is unidentified (and irrelevant) under period-by-period bilateral contracting, with multiperiod agreements, the parameter will play a key role in how future states are translated to starting prices. This association provides a mechanism that can be used to separately identify β from flow payments.

Unfortunately, with multiperiod agreements, even traditional bargaining parameters may not be identified. This is unsurprising given the lack of identification in dynamic discrete choice models (Rust, 1994).

Proposition 4 (Potential nonidentification and static representation with multiperiod contracts)**.** *Suppose $T \geq 2$, let $B^{(T)}$ be the set of structures generated by recursive Kalai models satisfying Assumption 2 with contract length T and such that $\ell^*(h) = 1$ for all (h) in the support of the game. Then $B^{(T)}$ is unidentified.*

Proof. Let data follow from some structure b with $\beta > 0$, $\varepsilon = 0$, and no uncertainty, so that d^* and Pay^* are deterministic conditional on x. Let b' be a structure with $\beta = 0$, $\tilde{Pay}(x) = E[Pay_t^* \mid x, \ell_t = T]$, and $\varepsilon = 0$. Then $b \Leftrightarrow b'$. □

This work provides sufficient conditions for identification. The arguments depend on an essential tool for dynamic recursive bargaining problems: the step-by-step property for static games.

3.2 The Step-by-Step Property for Recursive Bargaining

This subsection summarizes the step-by-step property's value for recursive problems. The key observation is that under Kalai proportional bargaining, gains from trade are always split in the ratio $\tau/(1-\tau)$. This means that when a negotiation is conducted relative to a single period of disagreement, one can replace the value of one disagreement with the value of repeated disagreement, canceling out many bargaining states. This cancellation arises from the Kalai proportional solution's step-by-step property (Dorn, 2026).

To illustrate the step-by-step property, consider Kalai proportional bargaining over two-period agreements with no unobservable utility ($\varepsilon = 0$), constant inflation ($\phi_t = 0$), and interior weights ($\tau \in (0,1)$).

The sides choose a starting payment $Pay^*(x)$ to satisfy the recursively defined equality:

$$\frac{\tau}{1-\tau} = \frac{V_1\left(Pay^*(x), x, \varepsilon = 0, \phi = 0, \ell = 2\right) - \beta E\left[V_1(Pay^*(x'), x', 0, 1, 2) \mid x\right]}{V_2\left(Pay^*(x), x, \varepsilon = 0, \phi = 0, \ell = 2\right) - \beta E\left[V_2(Pay^*(x'), x', 0, 1, 2) \mid x\right]}. \tag{3}$$

This is a complicated expression to work with: expectations over x' given x affect the bargaining outcome through the expected value of one-period disagreement followed by bargaining at some x'; the one-period disagreement bargain at x' is recursively defined in terms of the conditional expected value of two-period disagreement followed by bargaining at some x''.

The solution turns out to be the same if the negotiators replace the value of one-period disagreement with the value of two-period disagreement. Hypothetically, imagine a cognitively simpler model in which the sides bargain over a payment Pay^{**} subject to disagreeing for two periods and then returning to equilibrium. This disagreement point is implausible but useful: under two-period commitment, the associated function $Pay^{**}(x)$ would satisfy:

$$\frac{\tau}{1-\tau} = \frac{V_1\left(Pay^{**}(x), x, 0, 1, 2\right) - \beta^2 E\left[V_1(Pay^*(x''), x'', 0, 1, 2) \mid x\right]}{V_2\left(Pay^{**}(x), x, 0, 1, 2\right) - \beta^2 E\left[V_2(Pay^*(x''), x'', 0, 1, 2) \mid x\right]}. \tag{4}$$

This model would be much easier to solve, because $V_i(Pay, x, 0, 1, 2) = u_i(x) + \beta E[u_i(x') \mid x] + (1+\beta)(2i-3)Pay + \beta^2 E\left[V_i(Pay^*(x''), x'', 0, 1, 2) \mid x\right]$, yielding the two-period problem:

$$\frac{\tau}{1-\tau} = \frac{u_1(x) + \beta E[u_1(x') \mid x] - (1+\beta)Pay^*(x)}{u_2(x) + \beta E[u_2(x') \mid x] + (1+\beta)Pay^*(x)}. \tag{5}$$

Equation (5) is the problem that the players would solve if they formed a two-period agreement relative to two-period disagreement in a two-period world. Intuitively, this cognitively simpler model imposes a disagreement rule that exhibits a finite-dependence-type cancellation of outcomes beginning in the third period. This finite dependence cancellation allows me to solve the outcome in closed-form:

$$(1+\beta)Pay^*(X) = (1-\tau)\left(u_1(x) + \beta E[u_1(x') \mid x]\right) - \tau\left(u_2(x) + \beta E[u_2(x') \mid x]\right). \quad (6)$$

This equivalence will follow even if the sides are forward-looking and follow single-period disagreement.

The equivalence of Pay^* and Pay^{**} under Kalai proportional bargaining is as follows. Write $\gamma_i(x)$ for the (possibly random) gains from trade achieved by negotiation at some state h. By Equation (3), $\gamma_1(x) = \frac{\tau}{1-\tau}\gamma_2(x)$ for all x. Then, by expanding Equation (3) and applying the law of iterated expectations,

$$\frac{\tau}{1-\tau} = \frac{V_1\left(Pay^*(x), x, 0, 1, 2\right) + \beta\frac{\tau}{1-\tau}E\left[\gamma_2(x) \mid x\right] - \beta^2 E\left[V_1(Pay^*(x''), x'', 0, 1, 2) \mid x\right]}{V_2\left(Pay^*(x), x, 0, 1, 2\right) + \beta E\left[\gamma_2(x) \mid x\right] - \beta^2 E\left[V_2(Pay^*(x''), x'', 0, 1, 2) \mid x\right]}.$$

By algebra that I omit for brevity, this constraint is the same as if the future gains were set to zero, which yields the two-period-disagreement problem in Equation (4) and the two-period-disagreement outcome in Equation (6).

I now leverage this property to prove sufficient conditions for nonparametric identification with multiperiod agreements.

3.3 Identification of Kalai with Multiperiod Agreements

I provide two alternative sufficient conditions for identification.

The first sufficient condition for identification is based on a remark by Rust (1994), who writes that in dynamic discrete choice models with no observed components of utility, some partial identification may come from "agents who make different choices in the same state." In recursive Kalai bargaining games, I show that this condition permits nonparametric identification.

Proposition 5 (Identification without unobservables)**.** *Let $B^{(T,1)}$ be the set of $b \in B^{(T)}$ such* *that $n > 1$, there are no unobserved utility shocks ($\varepsilon = 0$), there is no inflation ($\phi = 0$), and there are at least $1 + T(n-1)$ combinations of (Pay, x) in the support of the data. Then if $T > 1$ and appropriate rank conditions hold (Appendix A.1), all bargaining parameters are identified under $B^{(T,1)}$.*

Intuitively, if there are no unobserved utility shocks, then variation in prices conditional on the current state must come from unobserved information on future states. By observing how the starting price Pay^* is correlated with future realized trajectories, the researcher can infer a discount factor and state-dependent flow payments.

Proof. See Appendix A.2 □

Proposition 5 implies that only some dynamic structures have a static representation as in Proposition 4. For example, payment variation that reflects information should not be mapped to a static utility shock, because the resulting utility shocks would need to be correlated with future information. Note also that $1 + T(n-1)$ is an unnecessarily large number of information states, and in fact would suffice to identify more flexible future-period discounting factors via ordinary least squares.

The second sufficient condition is motivated by empirical practice, which tends to favor GMM estimation. I show that an instrument satisfying an exclusion restriction can suffice to identify the discounting factor.

Proposition 6 (Identification from inflation expectations)**.** *Suppose $T > 1$, payments are strictly positive almost surely, and ζ_t is a real-valued observed coarsening of i_t satisfying the exclusion restriction $\{x_t, \varepsilon_t\}_{t=t_0}^{t_0+T-1} \perp\!\!\!\perp \zeta_{t_0} \mid x_{t_0}, \phi_{t_0}$ and the relevance condition that for all $t = 1, \ldots, T-1$, $E\left[\prod_{0<s\leq t}(1+\phi_{t_0+s}) \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0}\right]$ is strictly increasing in ζ_{t_0}. Then if there are at least $n + (T-1)$ values of (x, ϕ, ζ) in the support such that appropriate rank conditions hold (Appendix A.1), all bargaining parameters are identified under $B^{(T)}$.*

Proof. More generally, identification follows from payments that are constructed based on a moment on any time weighting over payments and gains. See Appendix A.2 for details. □

I view this second condition as more plausible in empirical use. The key restriction is that ζ_t affects bargaining only through expectations about inflation, and there is a clear direction of payments so that inflation expectations affect starting payments. Empirical practice also can call for interactions across negotiations by different players.

4 Conclusion

This paper provides a characterization of identification in the presence of unobserved information timing. The central finding is that among bargaining solutions satisfying independence of irrelevant alternatives, only the family of Kalai proportional solutions is identified in general. I show that identification implies concavity; I adapt results from Myerson (1981) to show that concavity implies utilitarian or proportional bargaining; and I note that utilitarian solutions are not identified, leaving only the family of Kalai proportional solutions. These results complement the known monotonicity and step-by-step benefits of the Kalai proportional solution for dynamic problems.

The practical content of this result is that Kalai proportional bargaining weights can be identified from a moment on expected gains from trade. Under Kalai proportional bargaining, the ratio of expected gains from trade must equal the ratio of bargaining weights, regardless of whether negotiation occurs ex-ante, ex-post, or somewhere in between. This moment gives the researcher a tool for identification and estimation without specifying information timing. Nash bargaining lacks such a moment; its axiomatic property of scale invariance, which is a necessary condition for microfoundations that use von Neumann-Morgenstern utility alone, implies that the act of comparing utility scales across states of the world will be substantive. As a result, the researcher's stance on information timing becomes an identifying assumption that can change the implied bargaining weights.

I show that even though Kalai proportional bargaining has a robust moment, bargaining weights may be unidentified with multiperiod contracts. I provide two sufficient conditions to restore identification: first, sufficient variation in agreements conditional on observed states, or alternatively, an instrument that only affects starting prices through observable price dynamics. This characterization leverages the step-by-step property of Kalai proportional bargaining to simplify the recursive bargaining problem. The companion paper Dorn (2026) develops the step-by-step property in detail and applies it to data on hospital–insurer contracting using an instrument inspired by the second identification condition.

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A Additional Content

A.1 Rank Conditions

Assumption 3 (Rank condition, Proposition 5)**.** Fix some x_0. The distinct values of (Pay, x) are such that the set of $P(X_{t_0+t} = x' \mid x_{t_0} = x, Pay_{t_0} = Pay)$ for initial periods t_0, $t = 0, \dots, T-1$, and $x' \neq x_0$ and a constant are linearly independent.

This rank condition is chosen to emphasize simplicity over comprehensiveness. For example, when $T > 1$, this condition requires i_{t_0} to be potentially informative for x_{t_0+2} given (x_{t_0}, x_{t_0+1}). This rank condition is sufficient to identify period-dependent weights β_t via ordinary least squares, which imply the fixed per-period discounting factor β.

Assumption 4 (Rank condition, Proposition 6)**.** For each (x, ϕ) combination in the tuples, pick some $\zeta_0(x, \phi)$ such that $(x, \phi, \zeta_0(x, \phi))$ is a tuple in the support. Then (i) consider the set of (x, ϕ, ζ) tuples $\zeta \neq \zeta_0(x, \phi)$. Then the set of values of

$$\left\{ \begin{array}{rl} & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta\right] \\ - & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta_0(x,\phi)\right] \end{array} \right\}$$

are linearly independent. Further (ii) let Σ be the matrix whose rows correspond to values of x and whose columns correspond to $\sum_{t=t_0}^{T-1} \beta^t P(x_{t_0+t} = x' \mid x_{t_0} = x)$. Then Σ is invertible.

This assumption is also chosen to simplify the resulting identification argument.

A.2 Additional Proofs

Proof of Lemma 1. Let $G \in \mathcal{G}$ be given. For v on the Pareto frontier of V, write $Pay(v) = \frac{v_2 - v_2^D}{\bar{v}_2 - v_2^D}$. Then $v_2^* + v_1^* = v_2^D + v_1^D + v$ and

$$\begin{aligned} &\tau v_1'\left(Pay\left(f_\tau^{(Nash)}(V, v^D)\right)\right)(v_2^* - v_2^D) = \frac{\tau(f_\tau^{(Nash)}(V, v^D)_2 - v_2^D)}{\bar{v}_2 - v_2^D} \\ = &(1-\tau)v_2'\left(Pay\left(f_\tau^{(Nash)}(V, v^D)\right)\right)(v_1^* - v_1^D) = \frac{(1-\tau)(f_\tau^{(Nash)}(V, v^D)_1 - v_1^D)}{\bar{v}_2 - v_2^D}, \end{aligned}$$

so that $\tau(v_2^* - v_2^D) = (1-\tau)(v_1^D + v - v_2^* - v_1^D)$. But $f_\tau^{(Kalai)}$ satisfies the same constraint. Therefore $f_\tau^{(Nash)}(V, v^D) = f_\tau^{(Kalai)}(V, v^D)$. □

Proof of Lemma 3. f is not concave, so there exists an V, W, λ such that at least one player strictly prefers $\lambda f(V, v^D) + (1-\lambda) f(W, u^D)$ to $f(\lambda V + (1-\lambda)W, \lambda v^D + (1-\lambda)u^D)$. By strict inequality, it must be that $\lambda \in (0, 1)$.

Without loss of generality, assume player 1 strictly prefers the ex post game. By WPO of the ex ante solution, player 2 weakly prefers the ex ante game. Write $v_{EA} = f\left(\lambda V + (1-\lambda)W, \lambda v^D + (1-\lambda)u^D\right)$, $v^* = f(V, v^D)$, and $w^* = f(W, u^D)$, then $v_{EA,1} < \lambda v_1^* + (1-\lambda)w_1^*$ and $v_{EA,2} \geq \lambda v_2^* + (1-\lambda)w_2^*$. A quirk is that the ex post choice may not be an ex ante outcome under any bargaining solution, so I produce a modified pair of games for which the ex post outcome is Pareto efficient (and so can be chosen) and the ex ante prediction under f is unchanged by IIA.

Let v', w' be points in V, W such that $v' \geq v^D$, $w' \geq u^D$, $v'_1 \geq v^*_1$, $v'_2 \leq v^*_2$, $w'_1 \geq v^*_1$, $w'_2 \leq w^*_2$, and $\lambda v' + (1-\lambda)w' = v_{EA}$. Such a point must exist because $v_{EA} \in \lambda V + (1-\lambda)W$.

Recall that the comprehensive convex hull of $\{v_1, \ldots, v_k\}$ is the set of $v \in \mathbb{R}^2$ such that $v \leq \sum_j \lambda_j v_j$ for some nonnegative λ_j summing to one (Myerson, 1981).

Now let V' be the comprehensive convex hull of v', v^*, and v^D, and let W' be the comprehensive convex hull of w', w^*, and u^D. The Pareto frontier of $\lambda V' + (1-\lambda)W'$ is the convex hull of $\lambda v' + (1-\lambda)w'$ and $\lambda v^* + (1-\lambda)w^*$. Let $X \geq 1/2$ correspond to playing V' and let $X < 1/2$ correspond to playing W'. Also let the utility functions u corresponding to mapping $p \in [0,1]$ to points on the utility frontier, with $p = 1/3$ corresponding to v' and w' and $p = 2/3$ corresponding to v^* and p^*. (Note that it is possible for the frontier of utility to be a single point in one of these games.) Also let $\mathscr{P}$ be the singleton distribution of *Bernoulli*(λ).

By the span constraint of Lemma 3, there exists an f' such that $f'(\lambda V' + (1-\lambda)W', \lambda v^D + (1-\lambda)u^D) = \lambda v' + (1-\lambda)w'$. Let b be a structure generated by $EA = 0$ bargaining under f, and let b' be a structure generated by $EA = 1$ bargaining under f'. Any equilibrium results in setting $p^* = 1/3$, so that $b \Leftrightarrow b'$. But f predicts setting $p^* = 2/3$ in the ex ante game, so that $f \not\Leftrightarrow f'$. Thus, $\mathcal{F}$ is not single-period identified. □

Proof of Lemma 4. For this proof, let $\mathcal{G}$ be the set of games (V, v^D) satisfying convexity and comprehensiveness, $v^D = 0$, and $V = \{v \in V : v \geq 0\}$. I write that the game $(\{0\}, 0) \in \mathcal{G}$, with $f(\{0\}, 0) = 0$ for any bargaining solution f.

I prove the weaker claim that if f satisfies WPO, IIA, and concavity for games in $\mathcal{G}$, then either f is proportional or f it utilitarian.

The proof generally follows Myerson (1981), so I proceed assuming the reader has a copy on hand. Let $\mathcal{G}$ be the set of games that satisfy Definition 1 and do not allow ex post losses, and let f be a bargaining solution that satisfies WPO, IIA, and this proof's weaker form of concavity. Myerson's Theorem 1 shows that because $\mathcal{G}$ is a convex combination, if f is linear, then f is utilitarian.

Most of Myerson's results follow after some change in notation. His Lemma 1 through Lemma 6 follow after adjusting the notion of comprehensive convex hull to intersect with $\mathbb{R}^2_+$, and writing $M = \{f(G) : G \in \mathcal{G}\}$. Lemma 7 follows after adding a caveat that the claim only holds if $\zeta = \lambda x(1-\lambda)y \geq 0$, and with some care verify the claim holds for $y = 0$.

Myerson's Lemma 8 requires more modification. Because f is not utilitarian, it continuous to be the case that there is a $v, u = f(v)$ such that $p \cdot v > p \cdot u$, where p is constructed in Myerson's Lemma 4. Then by Myerson's Lemma 7, take $\lambda = \frac{1}{\|u\|}$ and mix the game $H(v)$ with zero to obtain a game, with associated solution c satisfying $\|c\| = 1$. Without loss of generality I write that $u = c$. Then it clear by inspection that $u - c \in M$, because $f(\{0\}, 0) = 0 = u - c$.

Myerson's Lemmas 9 and 10 follow immediately. Lemma 11 follows with the caveat that the claim only holds if $u + \lambda d \geq 0$. Myerson's Lemma 12 completes the proof by showing that if $x \in M$, then $x = u + (p \cdot x - p \cdot u)\, u$, which holds so long as $u + (p \cdot x - p \cdot u)\, u \geq 0$. But by construction in this extension, $p \cdot u = 1$, so that the only requirement is that $p \cdot x \geq 0$. But $x \in M$, so there is an 0 such that $x = F(V, 0)$, and $0 \in M \cap V$, so that by Myerson's Lemma 5, $p \cdot x \geq p \cdot 0 = 0$. Therefore $u + (p \cdot x - p \cdot u)u \geq 0$, so that the result of Myerson's Lemma 12 holds and if f is not utilitarian, then f is proportional. □

Proof of Proposition 5. By Proposition 3, all bargaining parameters are identified if $T = 1$, so I proceed assuming $T > 1$.

By applying the step-by-step property T times and then canceling future (exogenously evolving) states, every (Pay, x) combination observed in a period t_0 must satisfy:

$$E\left[\sum_{t=0}^{T-1} \beta^t \prod_{0<s\leq t} (1 + \phi_{t_0+s}) \mid x_{t_0}\right] Pay = E\left[\sum_{t=0}^{T-1} \beta^t \tilde{Pay}(x_{t_0+t}) \mid x_{t_0}\right].$$

By assumption, there are at least $1 + T(n-1)$ pairs of (Pay, x) and (Pay', x) with

$Pay \neq Pay'$ both in the support of $Pay^* \mid x$. Then for each such pair:

$$Pay = \sum_{t=0}^{T-1} \sum_{x'} \beta^t \frac{1-\beta}{1-\beta^T} \tilde{Pay}(x') P(x_{t_0+t} = x' \mid x_{t_0} = x, Pay_{t_0} = Pay).$$

Fix some x_0. Then:

$$Pay = \overbrace{\tilde{Pay}(x)}^{b_x} + \overbrace{\sum_{t=0}^{T-1} \sum_{x' \neq x_0} \beta^t \frac{1-\beta}{1-\beta^T} \left(\tilde{Pay}(x') - \tilde{Pay}(x_0) \right)}^{c_{tx'}} P(x_{t_0+t} = x' \mid x_{t_0} = x, Pay_{t_0} = Pay).$$

Write the vector $\mathbf{p}$ of values of Pay, writing $\mathbb{X}$ as the matrix whose rows are each distinct (Pay, x) combination and whose columns are a constant one followed by one column for each of the probabilities. Then by the rank condition (Assumption 3) on the $1 + T(n-1)$ pairs, the values of $b_x = \tilde{Pay}(x)$ and $c_{t,x'}$ are identified for all $t \geq 0, x' \neq x_0$ by $(\mathbb{X}^T\mathbb{X})^{-1}\mathbb{X}^T\mathbf{p}$. Dividing $c_{1,x'}/c_{0,x'}$ for some $x' \neq x_0$, which exists by the assumption $n > 1$, identifies β. Then dividing $c_{t,x'}$ by $\beta^t(1-\beta)/(1-\beta^T)$ for any $x' \neq x_0$ identifies $\tilde{Pay}(x') - \tilde{Pay}(x_0)$. But $\tilde{Pay}(x_0)$ is already identified, so all bargaining parameters are identified. □

Proof of Proposition 6. Consider the distribution in new-contract periods t_0. Condition on a pair of x_{t_0}, ϕ_{t_0} in the support. By applying the step-by-step property T times, canceling future (exogenously evolving) states, writing $\beta_t = \beta^t$, and applying iterated expectations, for every $(x_{t_0}, \phi_{t_0}, \zeta)$ in the support, the parameters must satisfy:

$$\begin{aligned} E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq T} (1+\phi_{t_0+s}) \right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta \right] &= E\left[\sum_{t=t_0}^{t_0+T-1} \beta^{t-t_0} \tilde{Pay}(x_t) \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta \right] \\ &= E\left[\sum_{t=t_0}^{t_0+T-1} \beta^{t-t_0} \tilde{Pay}(x_t) \mid x_{t_0}, \phi_{t_0} \right]. \end{aligned}$$

Let ζ_0 be as in Assumption 4, i.e. $(x, \phi, \zeta_0(x, \phi))$ is in the support of (x, ϕ, ζ). Then for all

(x, ϕ, ζ) in the support:

$$\begin{aligned}
0 &= E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq t} (1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta\right] \\
&- E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq t} (1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta_0(x_{t_0}, \phi_{t_0})\right] \\
&= \sum_{t=0}^{T-1} \beta_t \left\{ \begin{array}{rl} & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta\right] \\ - & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta_0(x_{t_0}, \phi_{t_0})\right] \end{array} \right\}.
\end{aligned}$$

Then, writing $y_{x,\phi,\zeta} = E\left[Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta\right] - E\left[Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta_0(x, \phi)\right]$, I obtain

$$y_{x,\phi,\zeta} = \sum_{t=1}^{T-1} \beta_t \left\{ \begin{array}{rl} & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta\right] \\ - & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta_0(x, \phi)\right] \end{array} \right\}.$$

By the relevance condition (so that each difference in expectations is nonzero for all $\zeta \neq \zeta_0(x, \phi)$), the assumption that we have at least $T-1$ such pairs with $\zeta \neq \zeta_0(x, \phi)$, and the rank condition (Assumption 4(i)), the parameters β_t are identified for all $t \geq 1$ by least squares. Then the moments:

$$E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq t} (1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}\right] = E\left[\sum_{t=t_0}^{T-1} \beta_t \tilde{Pay}(x_t)\right]$$

identify $\tilde{Pay}(x_t)$ by the rank condition (Assumption 4(ii)) by writing $\mathbf{p}$ for the vector whose elements are $E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x\right]$, taking Σ as in Assumption 4, and identifying the vector of $\tilde{Pay}(x)$ values via $\Sigma^{-1}\mathbf{p}$. Thus, all bargaining parameters are identified. (In the net present value case, the discounting factor β is identified as β_1.) □